Spaces of valuations on quasi-Polish spaces

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Polish spaces (separable completely metrizable spaces) play a central role in measure theory. One nice property is that the space of Borel probability measures on a Polish space is again Polish when given the weak topology. This is very useful when constructing a measure in terms of a sequence of simpler approximating measures.

In theoretical computer science, in particular domain theory, it is more common to use (continuous) valuations instead of Borel measures, as they allow a more constructive approach to measure theory. Little is lost by this approach because each Borel measure uniquely determines a valuation, and the converse holds for most "nice" spaces in the sense that (locally finite) valuations can be extended to a (unique) Borel measure. It is known that the space of probabilistic valuations on a continuous domain is again a continuous domain.

The purpose of this talk is to provide support to the claim that quasi-Polish spaces provide a convenient framework for unifying these approaches to measure theory.

Quasi-Polish spaces are a class of countably based spaces that generalize both Polish spaces and ω-continuous domains. Quasi-Polish spaces can be defined as the countably based spaces that have a topology generated by a (Smyth-) complete quasi-metric, but they have many other equivalent characterizations.

In this talk, we will show that every locally finite continuous valuation on a quasi-Polish space extends uniquely to a Borel measure. We then show that the weak topology on the space of all (extended real valued) valuations on a quasi-Polish space yields another quasi-Polish space. We also obtain similar results for the spaces of probabilistic valuations and sub-probabilistic valuations.

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