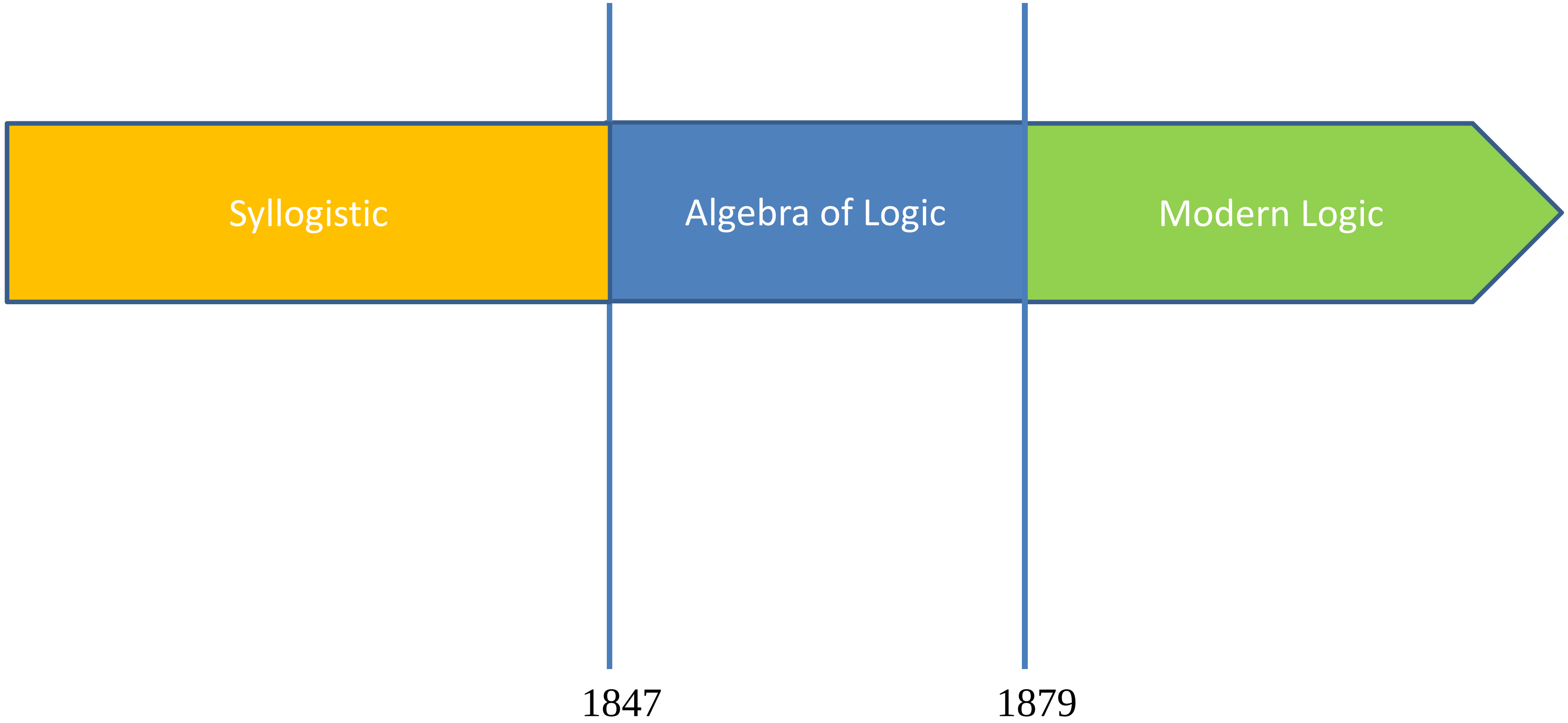
George Boole
and the generalisation of logical processes

Amirouche Moktefi

Chair of Philosophy

Tallinn University of Technology, Estonia

History of modern logic


Syllogistic
Algebra of Logic
Modern Logic
1847
1879

EUCLID

AND HIS

MODERN RIVALS

BY

CHARLES L. DODGSON, M.A.

Senior Student and Mathematical Lecturer
of Christ Church, Oxford

'All for your delight
We are not here. *That you should here repent you*
The actors are at hand; and, by their show,
You shall know all, that you are like to know.'

London
MACMILLAN AND CO.
1879

1. Th
$X \;\mathbf{P}\; T_{a''}$

2. Th
$X \;\mathbf{P}\; T_{e''\,i''}$

3. Th
$st_a \;\mathbf{P}\; S$

4. Th
$st_{ei} \;\mathbf{P}\; S$

8. Th
sS real

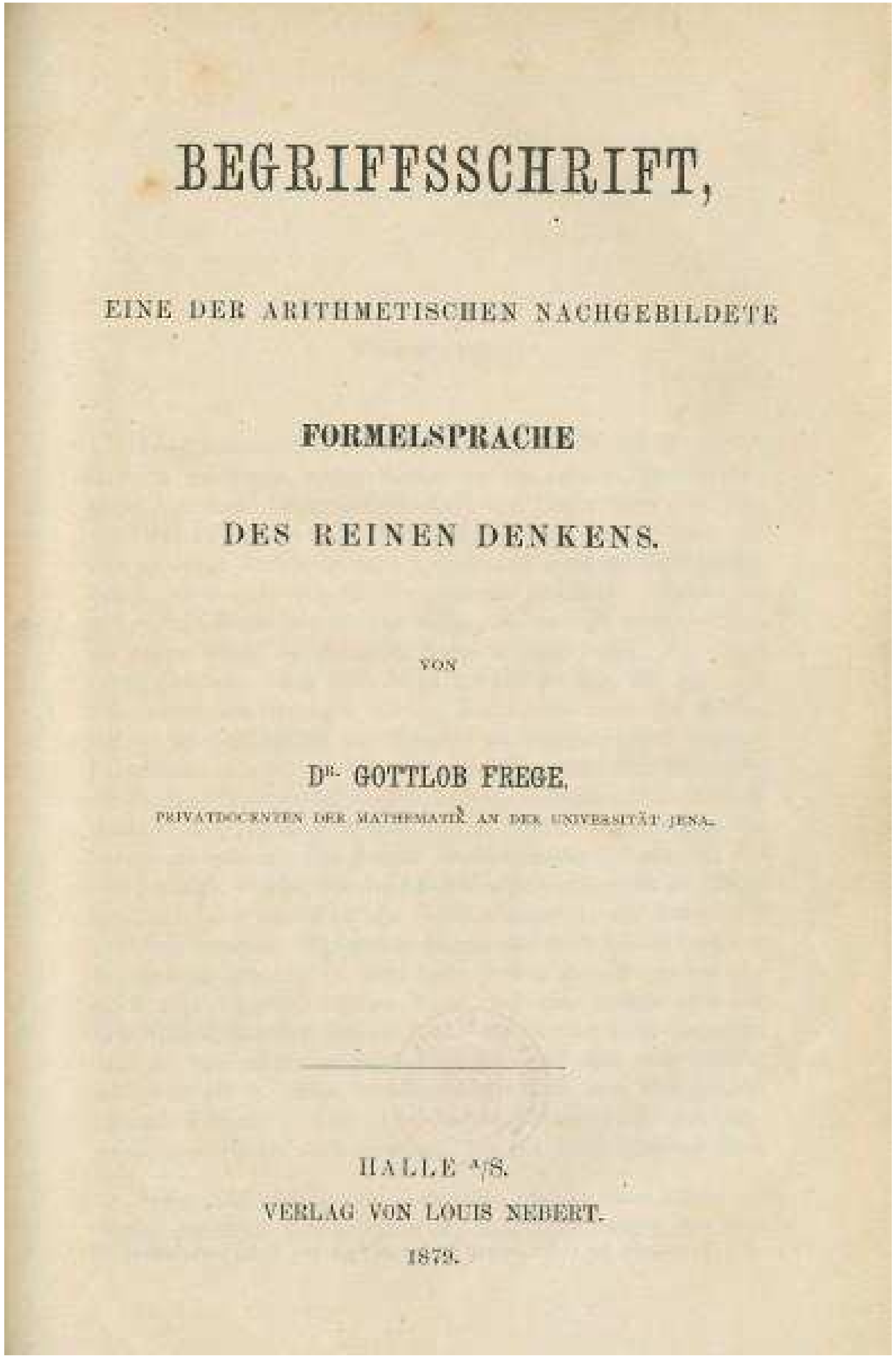
5. Ax
$st_{y} \;\mathbf{P}\; X$

6. Th
$S \;\mathbf{P}\; T_m$

7. Th
$\overline{s\,|\,S\,|\,S} \;\mathbf{P}\; \overline{S\,|\;\;|}$

9. Th
ext ∠ of Δ =
2 int opp ∠s

10. Th
3 ∠s of Δ = 2 ∟s



BEGRIFFSSCHRIFT,

EINE DER ARITHMETISCHEN NACHGEBILDETE

FORMELSPRACHE

DES REINEN DENKENS.

VON

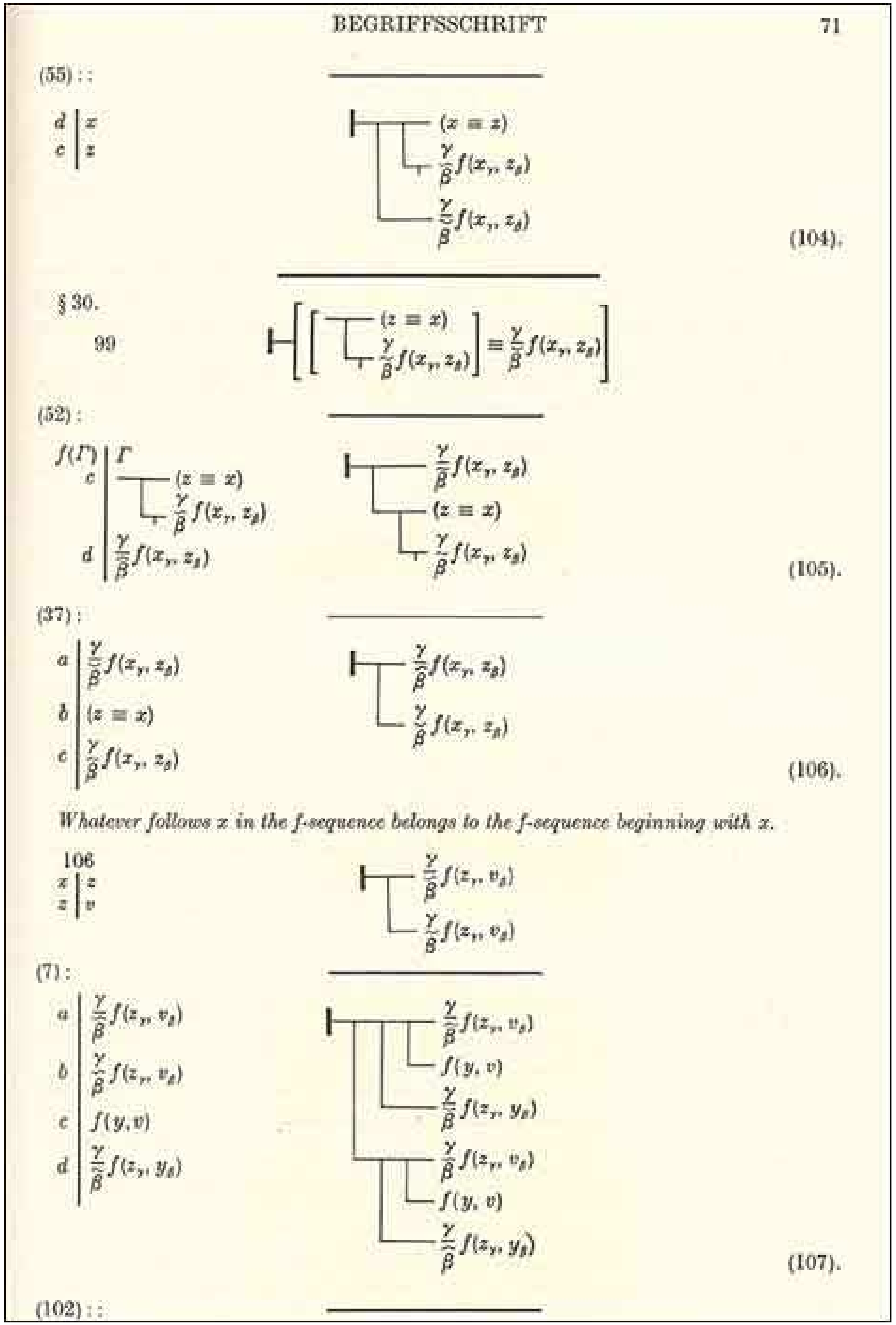
DR. GOTTLOB FREGE,

PRIVATDOCENTEN DER MATHEMATIK AN DER UNIVERSITÄT JENA.

HALLE a/S.

VERLAG VON LOUIS NEBERT.

1879.



(55) : :

(104).

§ 30.

99

(52) :

(105).

(37) :

(106).

Whatever follows x in the f-sequence belongs to the f-sequence beginning with x.

106

(107).

(102) : :

FROM FREGE TO GÖDEL

A Source Book in Mathematical Logic, 1879-1931

Jean van Heijenoort

PROFESSOR OF PHILOSOPHY, BRANDEIS UNIVERSITY

HARVARD UNIVERSITY PRESS

CAMBRIDGE, MASSACHUSETTS · 1967

‘Boole, De Morgan, and Jevons are regarded as the initiators of modern logic, and rightly so […] Considered by itself, the period would, no doubt, leave its mark upon the history of logic, but it would not count as a great epoch.

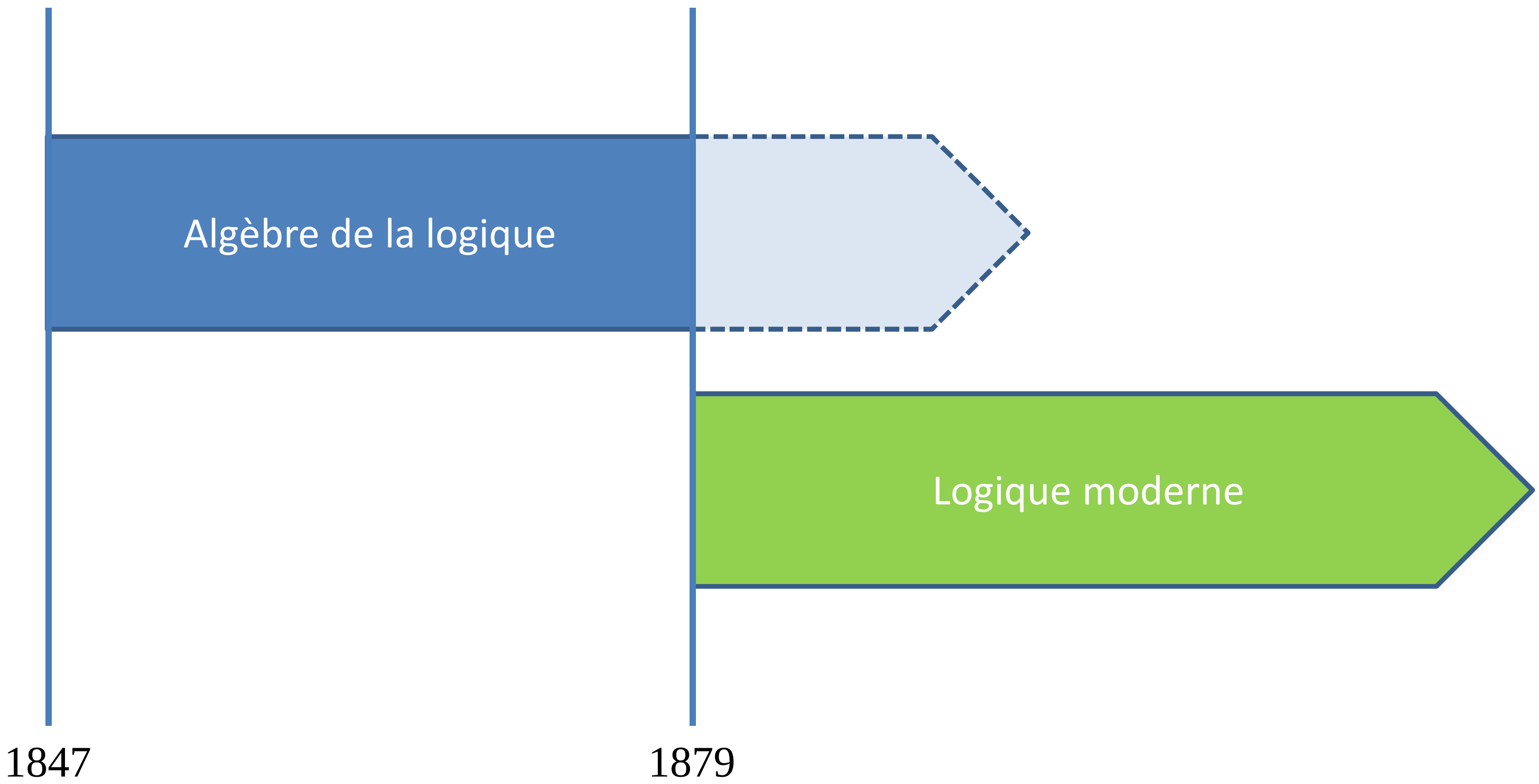
A great epoch in the history of logic did open in 1979, when Gottlob Frege’s *Begriffsschrift* was published.’

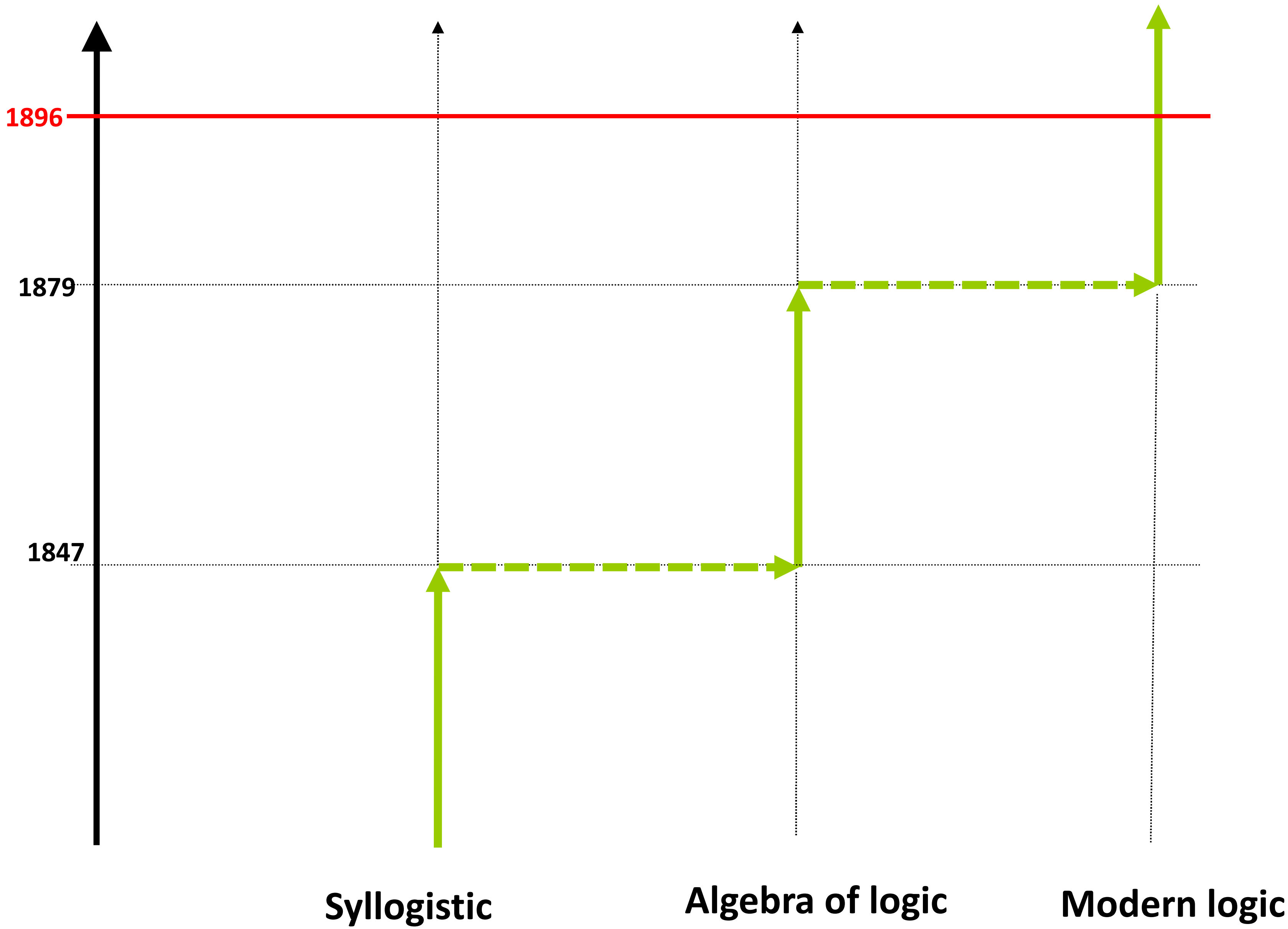
(*From Frege to Gödel*, 1967, vi)

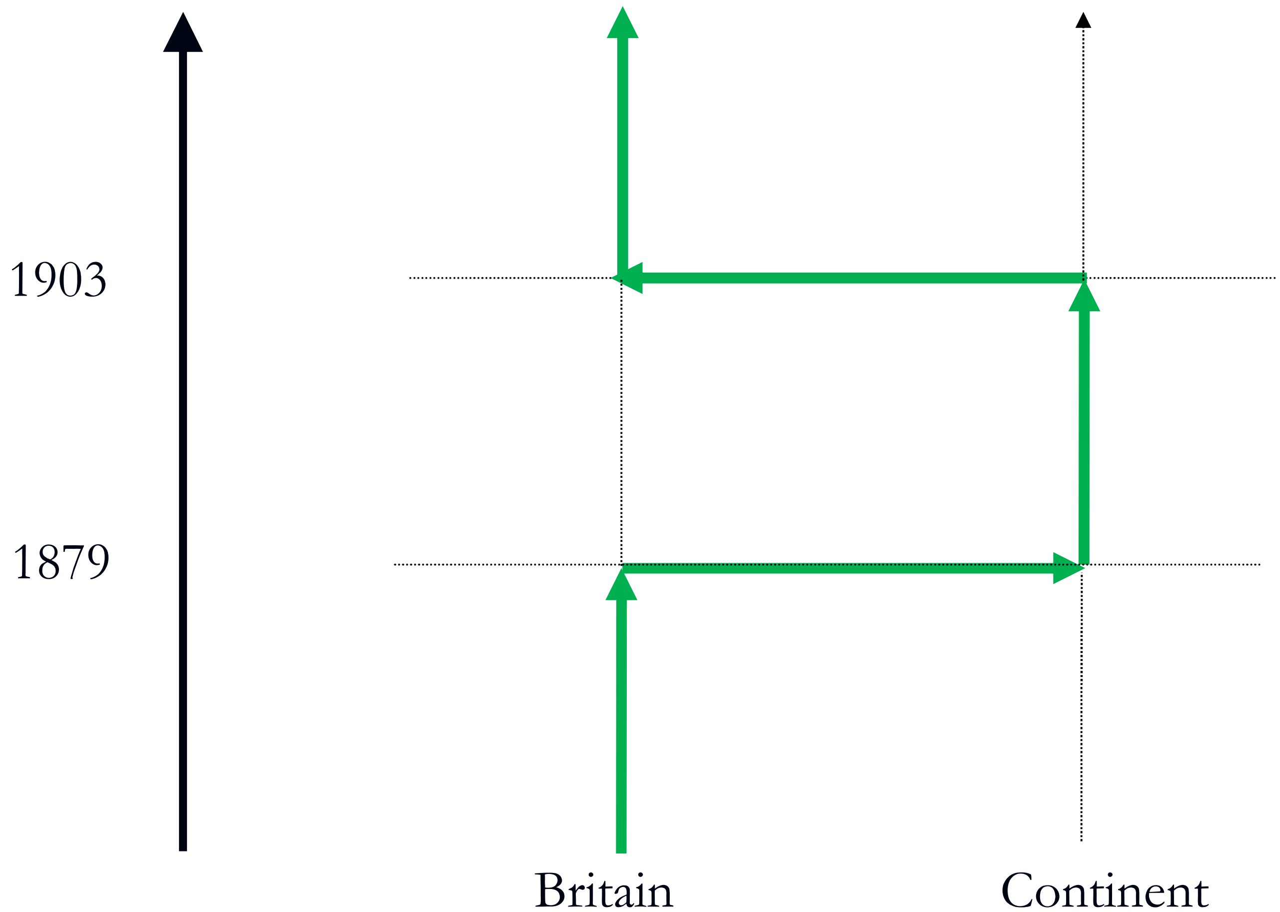
‘Modern Logic began in 1879, the year in which Gottlob Frege (1848-1925) published his Begriffsschrift. In less than ninety pages this booklet presented a number of discoveries that changed the face of logic.’

(‘Historical development of modern logic’, *Logica Universalis*, 2012, 327)

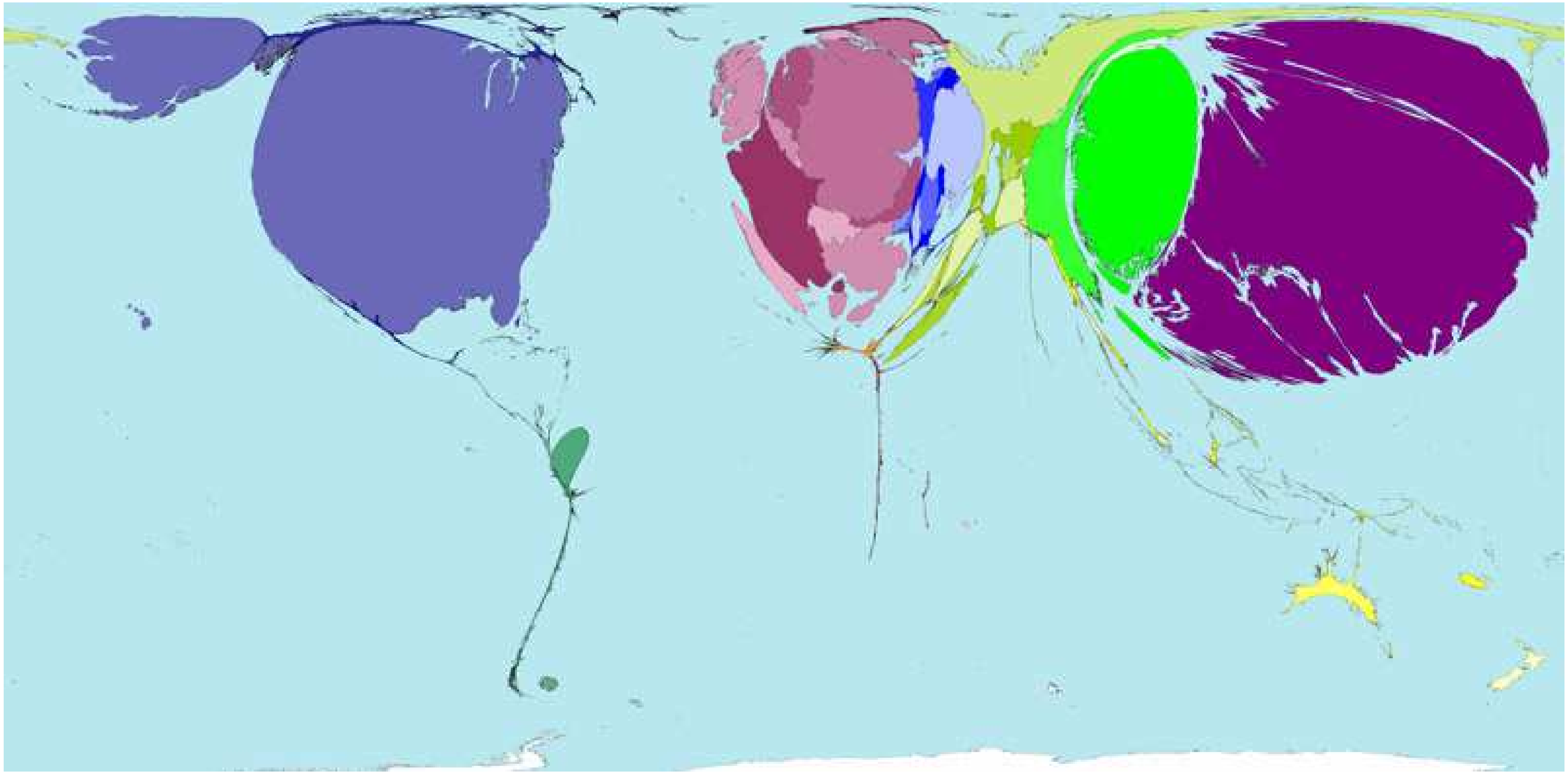
Logic as calculus and *logic as language*




1896
1879
1847
Syllogistic
Algebra of logic
Modern logic



1903
1879
Britain
Continent



To exist on the map, you have to be/look bigger

(Boole doesn't need that!)

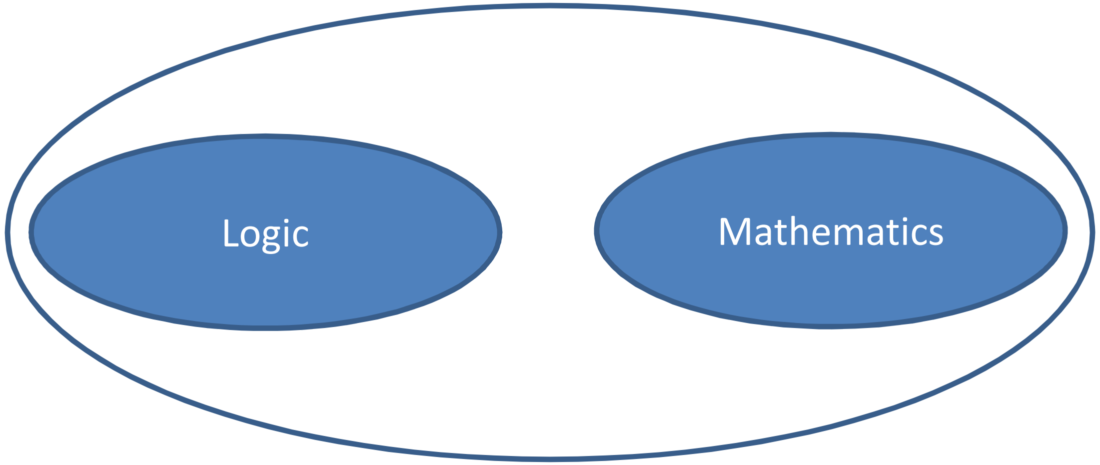
Methodological principle 1

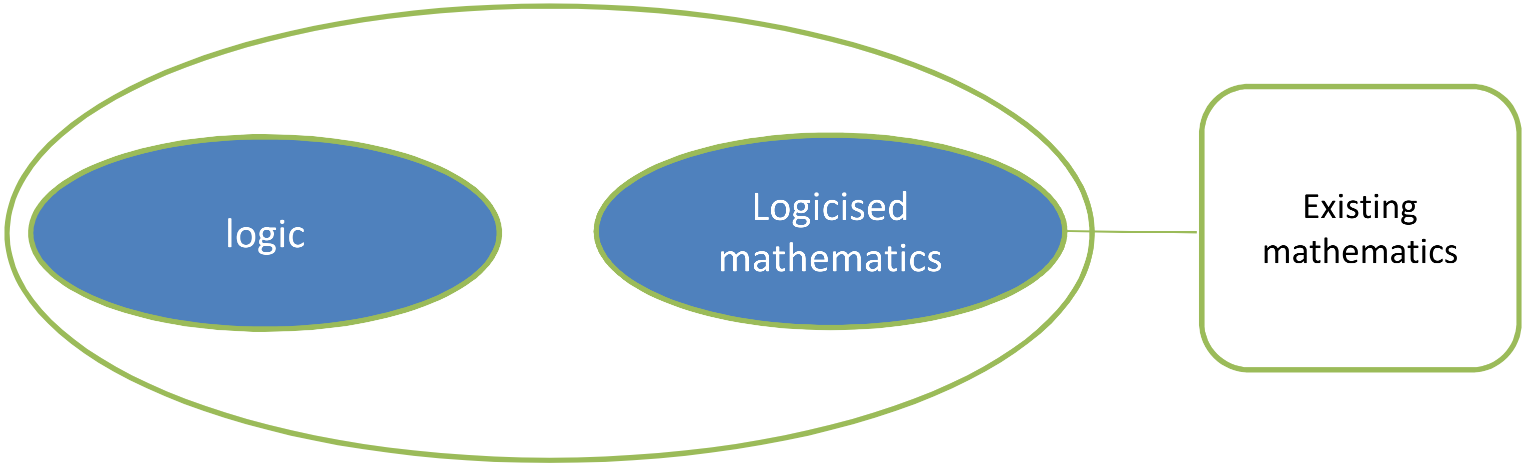
The History of the modernity of logic

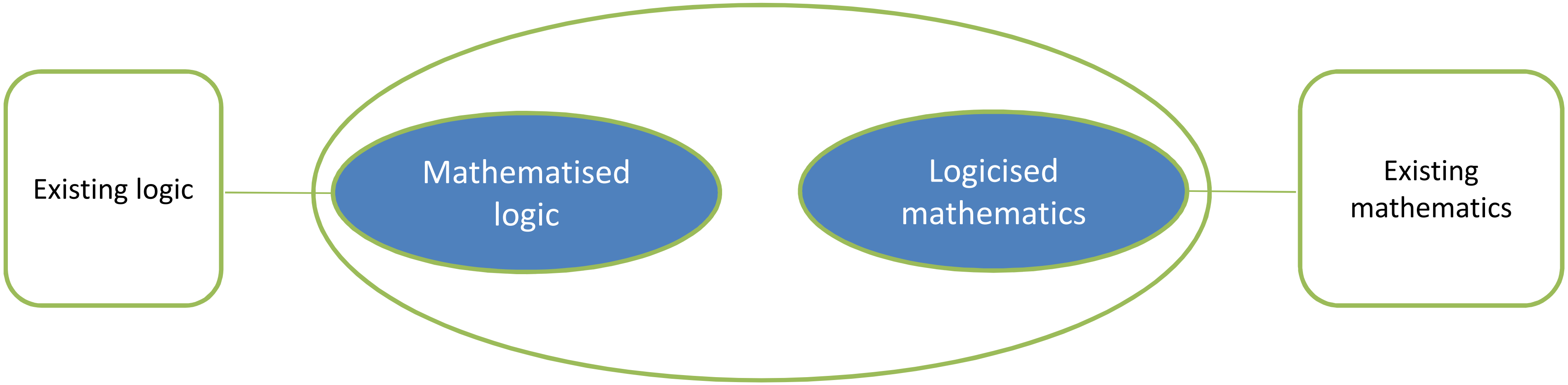
rather than

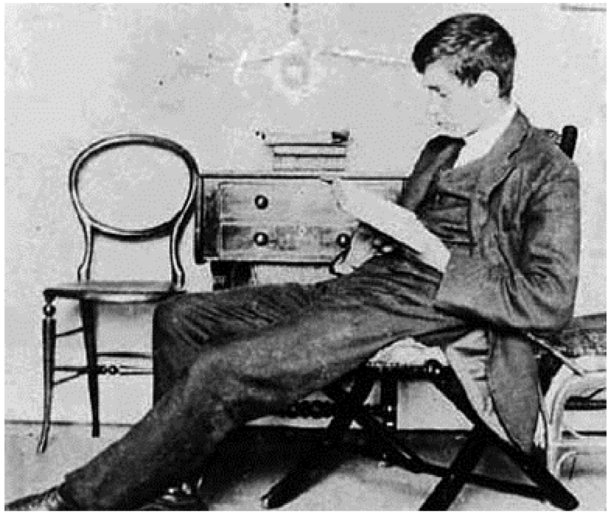
the history of modern logic

Logic *vs* Mathematics


Logic
Mathematics


logic
Logicised mathematics
Existing mathematics


Existing logic
Mathematised logic
Logicised mathematics
Existing mathematics

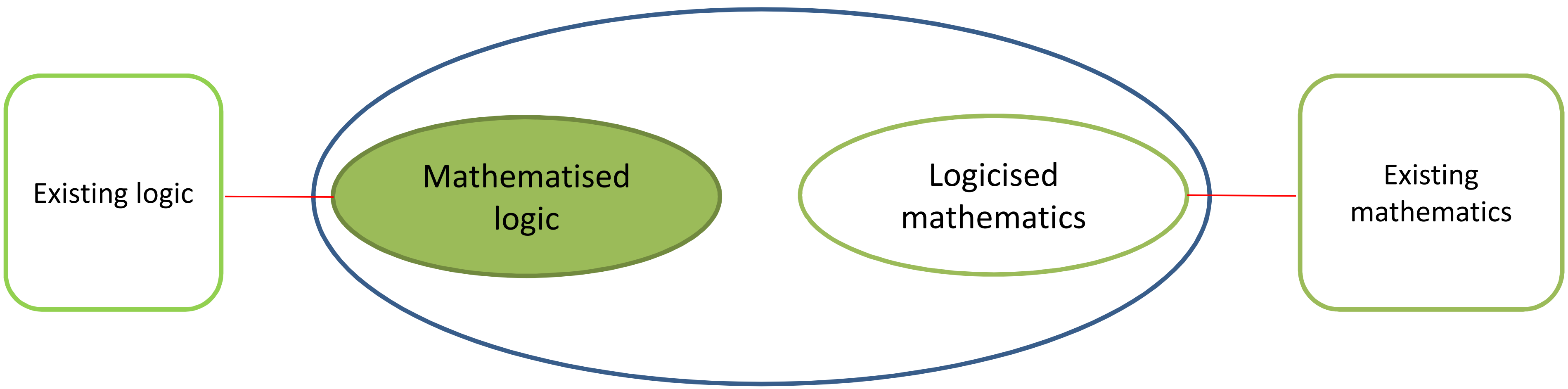


'Mathematical logic is a necessary preliminary to logical mathematics'

(P. E. B. Jourdain, *Preface* to Louis Couturat's *The Algebra of Logic*, 1914)

Mathematical logic *vs* logic of mathematics

Mathematical logic of mathematics



Existing logic
Mathematatised logic
Logicised mathematics
Existing mathematics

Methodological principle 2

The history of Mathematical logic as a mahematical discipline

rather than

The history of Mathematical logic as a foundation for mathematics

Mathematical logic?

“On the principle of a true classification, we ought no longer to associate Logic and Metaphysics, but Logic and mathematics” (Boole, *MAL*, 1847: 13)

“Logic as a Science is a branch of the larger science of Reasoning by Signs, another form of which is exhibited in ordinary Mathematics” (Boole, *Elementary treatise on logic not mathematical including philosophy of mathematical reasoning*, probably before 1849)

'The introduction of mathematical symbols and methods of working into logic is indeed, on every account, to be protested against by all who are interested in the welfare of the science. The rejection of these is the more to be insisted on, as well-meaning efforts still continue to be made to improve logic by mathematical treatment […] The notion of extending the sphere of mathematics so as to include logic, is as theoretically absurd as its realisation is practically impossible. To identify logic with mathematics is to make the whole equal to its part […] All such endeavours possess the singular merit of making logic as repulsive as possible, without doing the least service to mathematics.'

(Thomas Spencer Baynes, *New Analytic of Logical Forms*, 1850: 150-151)

'The forms of my system may, in fact, be reached by divesting [Boole's] system of a mathematical dress, which, to say the least, is not essential to it. The system being restored to its proper simplicity, it may be inferred, not that Logic is a part of Mathematics, as is almost implied in Prof. Boole's writings, but that the Mathematics are rather derivatives of Logic.'

(W. S. Jevons, *Pure Logic*, 1864: 3)

'I shall endeavor to show that there is nothing in Boole's logic which can properly be called mathematical; that it is simply a **generalization** of very familiar logical principles, with a certain necessary shifting, however, from the ordinary point of view […] whereas the common logic uses symbols for *classes*, and for hardly anything else, we shall make equal use of symbols *for operations upon these classes*... The fact that we do this, and still more that we find it convenient to adapt the peculiar symbols of mathematics to our purpose, has given rise to the impression (and unfounded one, as I hope to show) that such a system of logic is a branch of mathematics'

(J. Venn, 'Symbolic Logic', *Princeton Review*, 1880: 248-249)

Symbolic logic

is

logic *with* symbols

SYMBOLIC LOGIC

295

BY

JOHN VENN, M.A.,

FELLOW, AND LECTURER IN THE MORAL SCIENCES,

GONVILLE AND CAIUS COLLEGE, CAMBRIDGE.

"Sunt qui mathematicum vigorem extra ipsas scientias, quas vulgo mathematicas appellamus, locum habere non putant. Sed illi ignorant, idem esse mathematice scribere quod in forma, ut logici vocant, ratiocinari."

LEIBNITZ, *De vera methodo Philosophiæ et Theologiæ* (about 1690).

"Cave ne tibi imponant mathematici logici, qui splendidas suas figuras et algebráicos mæandros universale inventionis veri medium crepant."

RÜDIGER, *De sensu veri et falsi*, Lib. II. Cap. IV. § xi. (1722).

London:

MACMILLAN AND CO.

1881

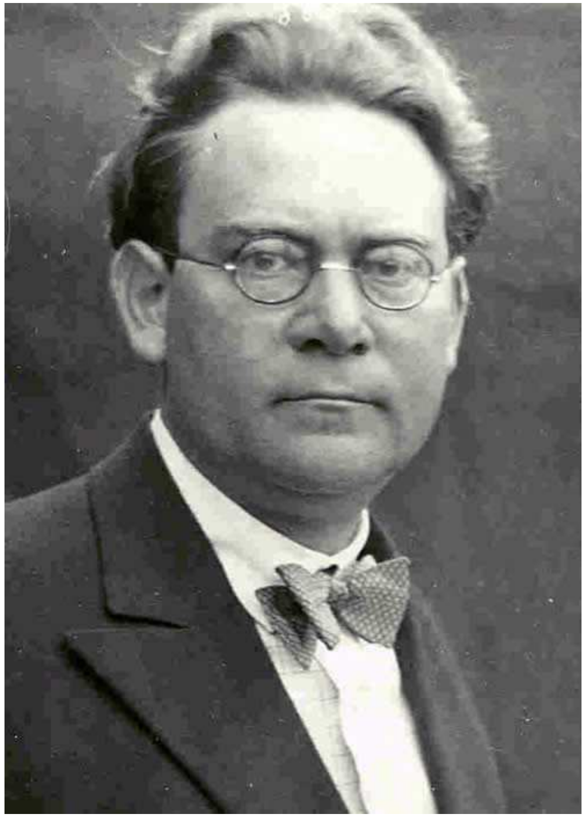
Symbolic Logic

- 1850: A. De Morgan, 'On the symbols of logic': 83
- 1854: G. Boole, *Laws of Thought*: 259 ['symbolical logic']
- 1877: W. S. Jevons, *The Principles of Science*: 423

- 1881: J. Venn, *Symbolic Logic*
- 1889: C. Ladd-Franklin, 'On some characteristics of Symbolic logic'
- 1896: L. Carroll, *Symbolic Logic*
- 1898: A. N. Whitehead, *Universal Algebra* [Book II: *The Algebra of Symbolic Logic*]
- 1903: B. Russell, The Principles of Mathematics [Chap. II: *Symbolic Logic*]
- 1906: H. MacColl, *Symbolic Logic and its Applications*
- 1906: A. T. Shearman, *The Development of Symbolic Logic*, 1906
- 1918: C. I. Lewis, *A Survey of Symbolic Logic*

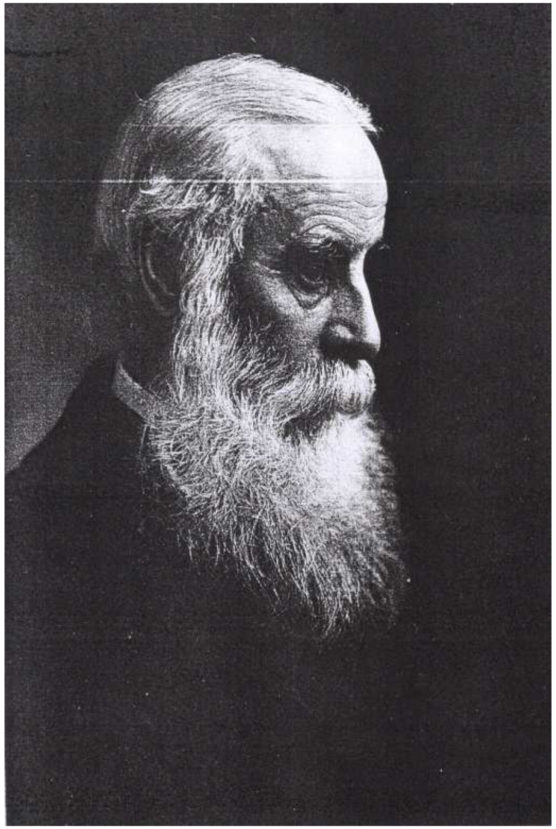
- 1936: *The Association for Symbolic Logic*

Who are symbolic logicians?



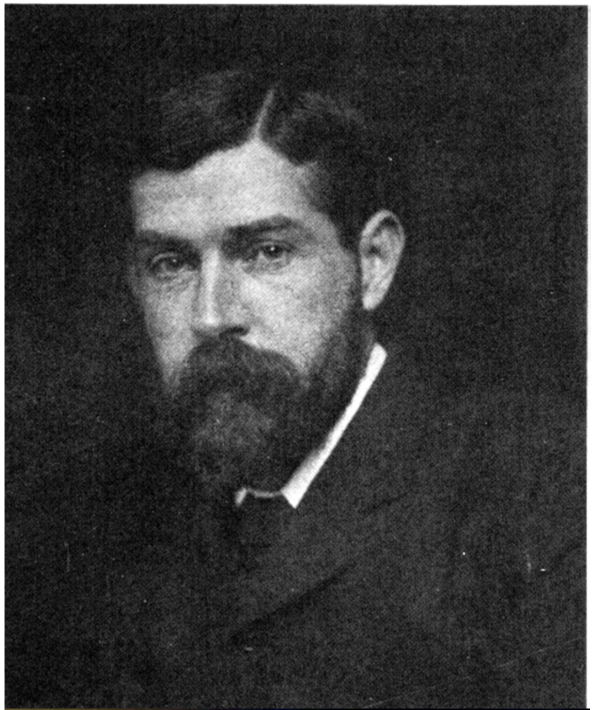
'Boole's work, from which we may date the modern period of logic, was already fifty years old. But **the new ideas had not yet acquired any significant publicity ; they were more or less the private property of a group of mathematicians** whose philosophical bias had led them astray into the realm of a mathematical logic. **The leading philosophers, or let us better say the men who occupied the chairs of philosophy, had not taken much notice of it and did not believe** that Aristotelian logic could ever be superseded, or **that a mathematical notation could improve logic**'

(H. Reichenbach, 'Bertrand Russell's logic', 1946: 24)



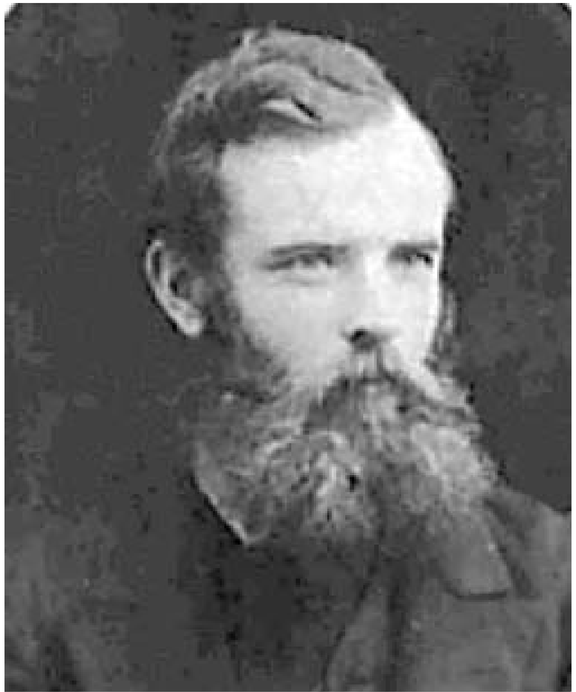
‘[S]ymbolic logic as such consists of a solution of particular problems, which are on the same plane as the solution of geometrical or algebraic problems and, though concerned with the abstract forms of subject and predicate, as specially scientific as these mathematical processes – **no more logic than they are, and related to logic precisely as they are.** Incidentally there is a little elementary logic involved, but the real and serious problems of logic proper do not appear, nor is the symbolic logician able to touch them. **In comparison with the serious business of logic proper, the occupations of the symbolic logician are merely trivial.’**

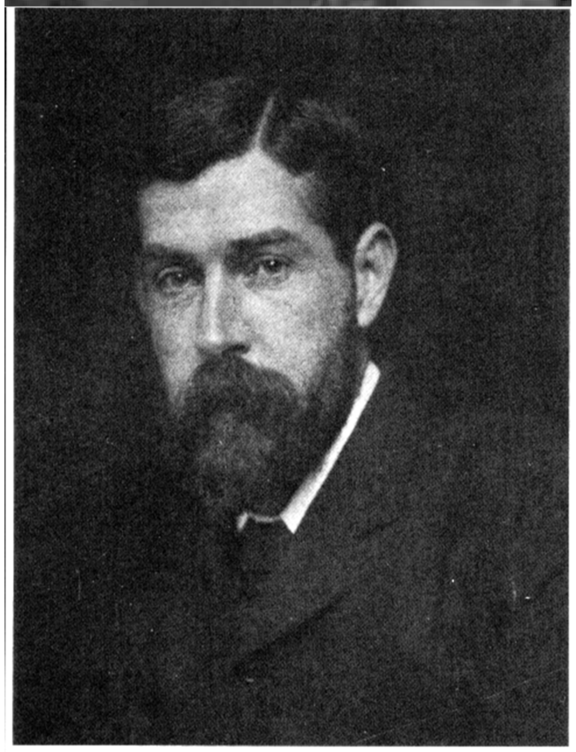
(J. C. Wilson, *Statement and Inference*, 1898/1926: 637)



‘If I were younger, I would at least make an effort to learn some mathematics, since to theorize on reasoning in general while ignorant of one branch of reasoning is obviously at best a precarious business. But it is too late now, even if it ever would have been possible for me. I really think I have no head for any abstract reasonings & should never be able to follow the processes of Symbolic Logic.’
(F. H. Bradley, *Letter to B. Russell*, 4 February 1904)

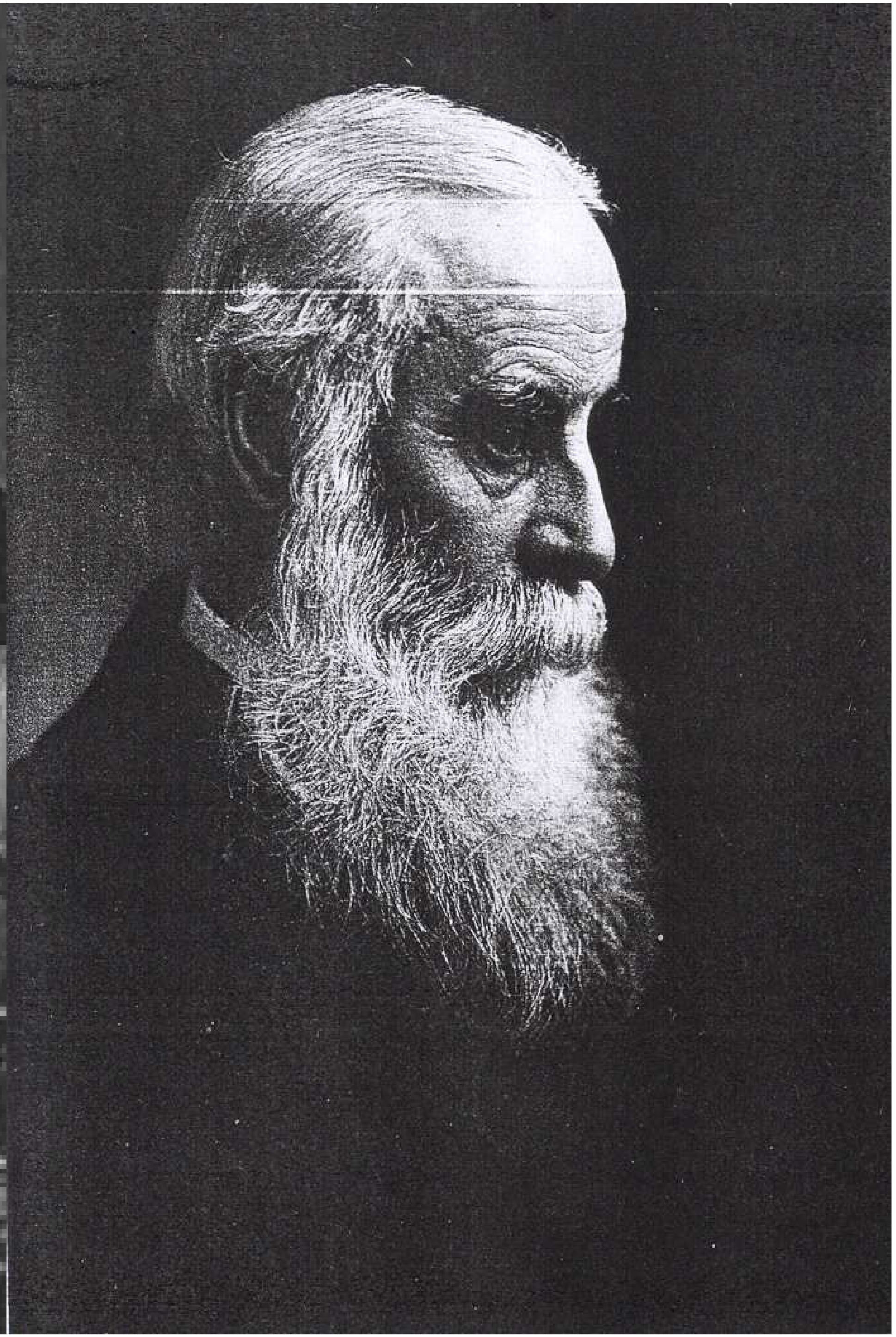
‘Mr MacColl […] is unintelligible to me. His symbols may be excellent for anything I know, but it seems to require a great many of them to expose an obvious mistake. Anyway I have no idea what he is saying & have no intention of learning his dialect, as time is more than short with me.’
(F. H. Bradley, *Letter to B. Russell*, 9 December 1904)





If I were a professor of logic, I would certainly get your books and study them; but as **I am only an amateur**, driven by I know not what mental perversity towards abstract studies from which I can never hope to reap any material gain or benefit, I am afraid I must content myself with the few books on logic that I already posses … I cannot afford the luxury of a large library.

(H. MacColl to F. H. Bradley, 14 December 1904)


Who cared about Symbolic Logic?

[T]his book is *not* offered as a "school book." In the present state of logical teaching, it has *no* chance of being "adopted" as "a school book," as it would be for no use in helping its readers to answer papers on the Formal logic, which is the only kind taught in Schools and Universities... **I have no doubt that Symbolic Logic (not necessarily *my* particular method, but *some* such method) will, *some* day, supersede Formal Logic, as it is immensely superior to it: but there are no signs, as yet, of such a revolution**.

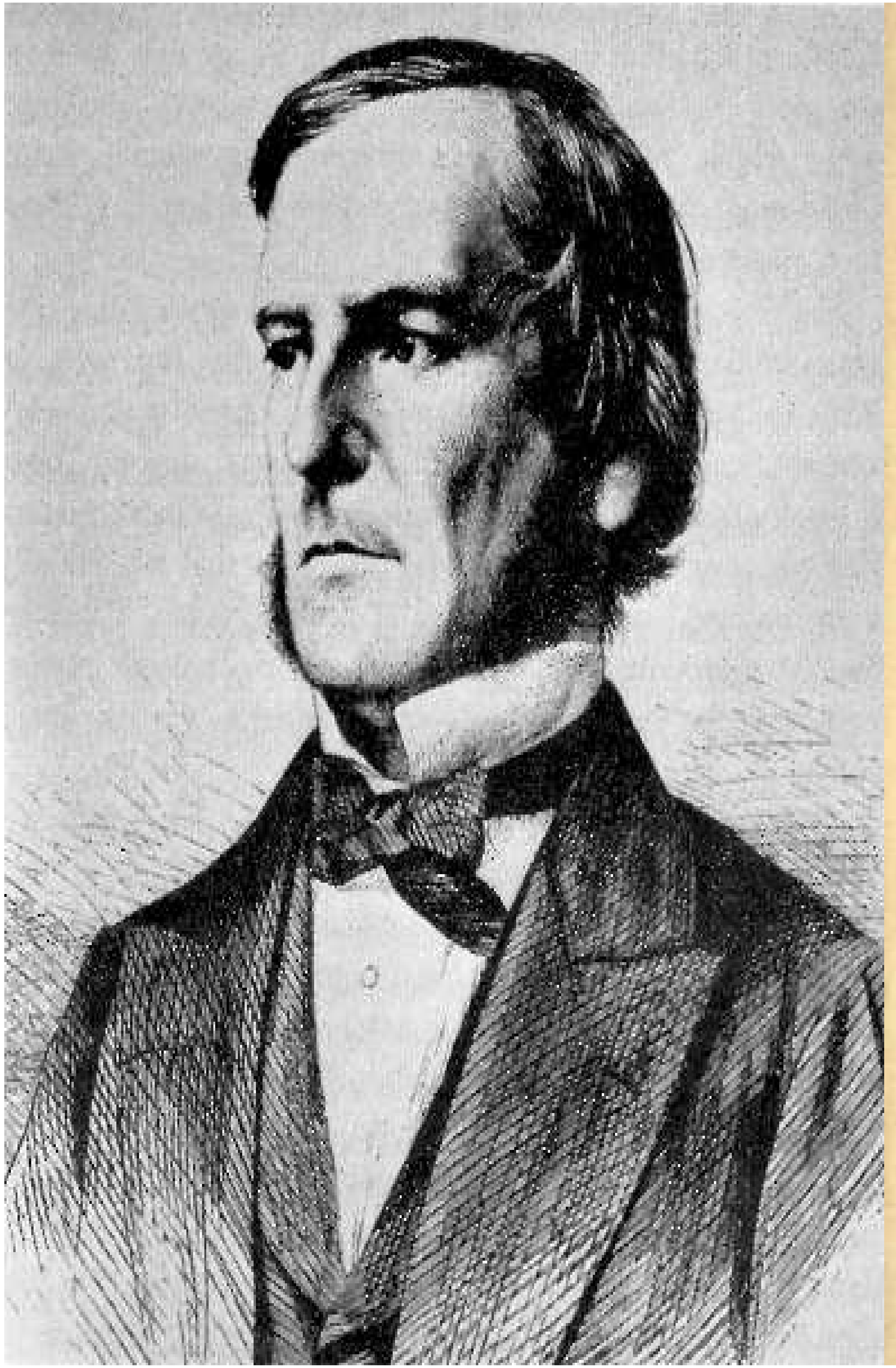
(L. Carroll, *Letter to Macmillan*, 19 October 1895)

- *The early years of Symbolic logic were not a success story*

- *Symbolic logicians were a minority of 'outsiders'*

- *By 1900, being a symbolic logician was not a job with prospects*

The Business of Symbolic Logic



AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

BY

GEORGE BOOLE, LL.D.

PROFESSOR OF MATHEMATICS IN QUEEN'S COLLEGE, CORK.

LONDON:
WALTON AND MABERLY,
UPPER GOWER-STREET, AND IVY-LANE, PATERNOSTER-ROW.
CAMBRIDGE: MACMILLAN AND CO.
1854.

Symbolic Logic (s)?

‘There are now **two systems of notation**, giving the same formal results, one of which gives them with self-evident force and meaning, the other by dark and symbolic processes. The burden of proof is shifted, and it must be for the author or supporters of the dark system to show that it is in some way superior to the evident system.’

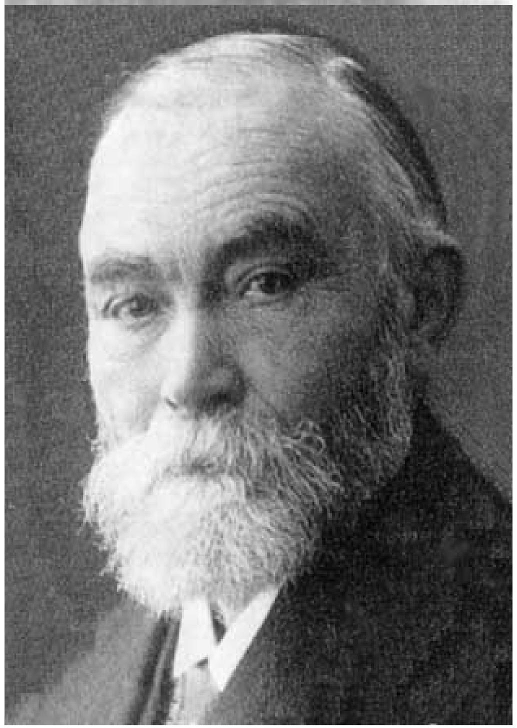
(Jevons, *Pure Logic*, 1864, p. 75)

‘**Four different algebraic** methods of solving problems in logic of non-relative terms have already been proposed by Boole, Jevons, Schröder, and MacColl. I propose here a fifth method which perhaps is simpler and certainly more natural than any of the others.’

(C. S. Peirce, ‘On the algebra of logic’, 1880: 37)

‘There are in existence five algebras of logic, - those of Boole, Jevons, Schröder, McColl, and Peirce, - of which the later ones are all modifications, more or less slight, of that of Boole. **I propose to add one more to the number**.’

(C. Ladd, ‘On the algebra of logic’, 1883: 17)



‘Dr. Frege’s work seems to be a somewhat novel kind of Symbolic Logic […] it does not seem to me that Dr. Frege’s scheme can for a moment compare with that of Boole. I should suppose, from his making no reference whatever to the latter, that he has not seen it, nor any of the modifications of it with which we are familiar here. Certainly the merits which he claims as novel for his own method are common to every symbolic method... I have not made myself sufficiently familiar with Dr. Frege’s system to attempt to work out problems by help of it, but I must confess that it seems to me cumbrous and inconvenient.’

(J. Venn, *Mind*, 1880: 297)

Syllogisms

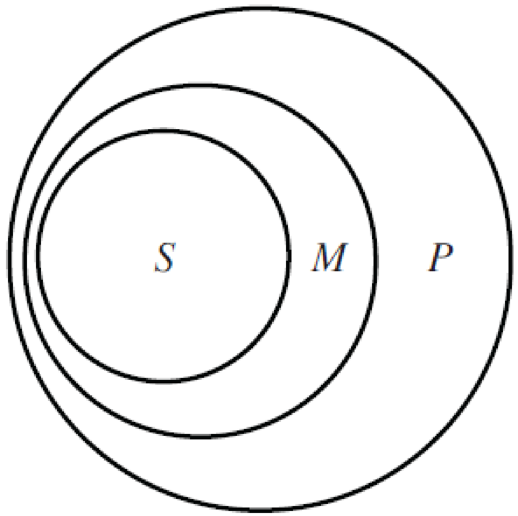
Every syllogism, then, consists of three propositions; the two first of which are called the *premises* and the third the *conclusion*. Now, the advantage of the all these [valid] forms to direct our reasoning is this, that if the premises are both true, the conclusion infallibly is so.

This is likewise the only method of discovering unknown truths. Every truth must always be the conclusion of a syllogism, whose premises are indubitably true. [Euler, 1833, p. 350]

All M are P

All S are M

All S are P



The Problem of Elimination

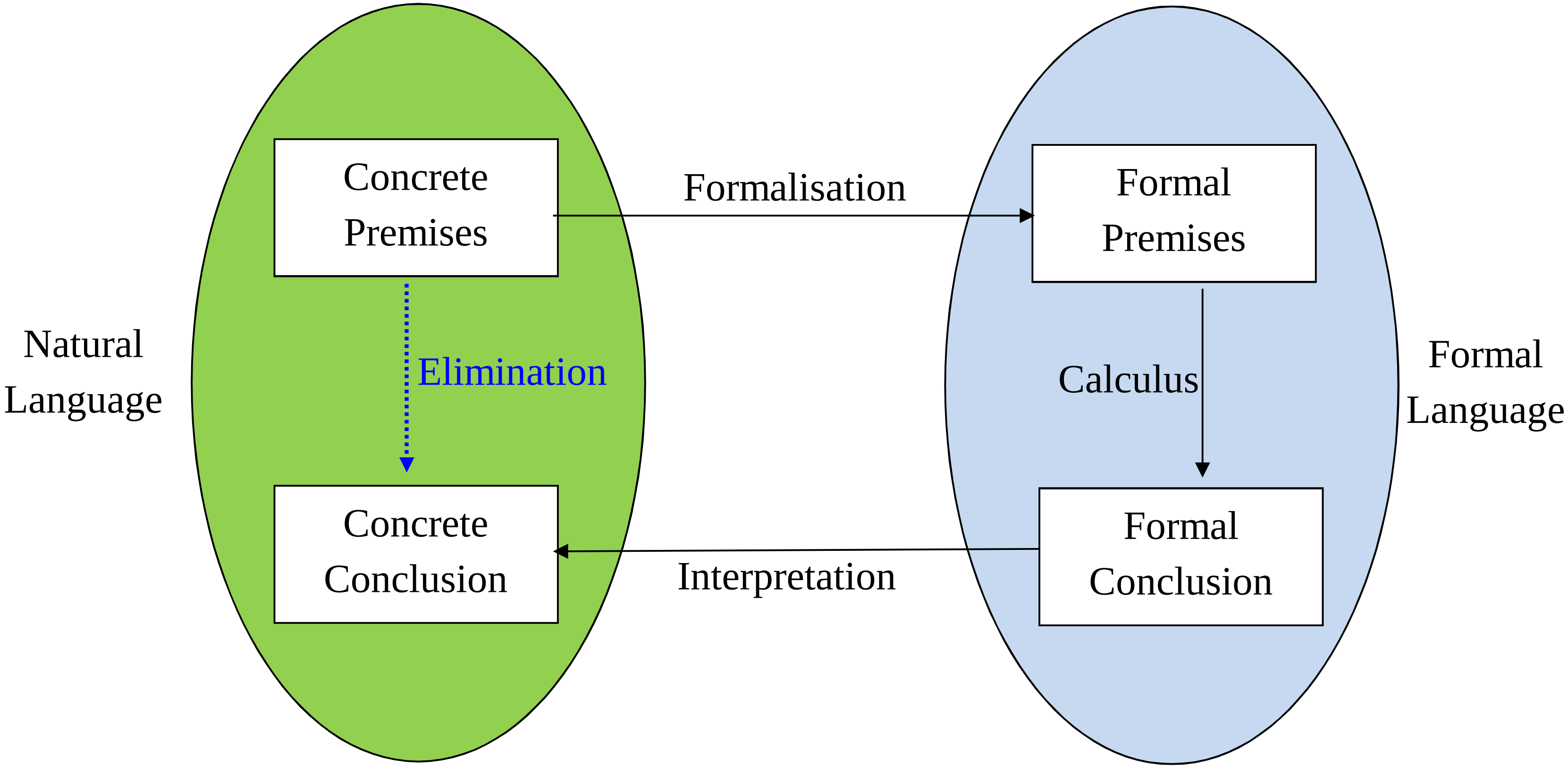
Introduced by Boole and considered as the main problem of logic by most symbolic logicians and *their opponents*

> As the conclusion must express a relation among the whole or among a part of the elements involved in the premises, it is requisite that we should possess the means of eliminating those elements which we desire not to appear in the conclusion, and of determining the whole amount of relation implied by the premises among the elements which we wish to retain. Those elements which do not present themselves in the conclusion are, in the language of the common Logic, called middle terms; and the species of elimination exemplified in treatises on Logic consists in deducing from two propositions, containing a common element or middle term, a conclusion connecting the two remaining terms. But the problem of elimination, as contemplated in this work, possesses a much wider scope. It proposes not merely the elimination of one middle term from two propositions, but the elimination generally of middle terms from propositions, without regard to the number of either of them, or to the nature of their connexion [Boole, 1854, p. 8]

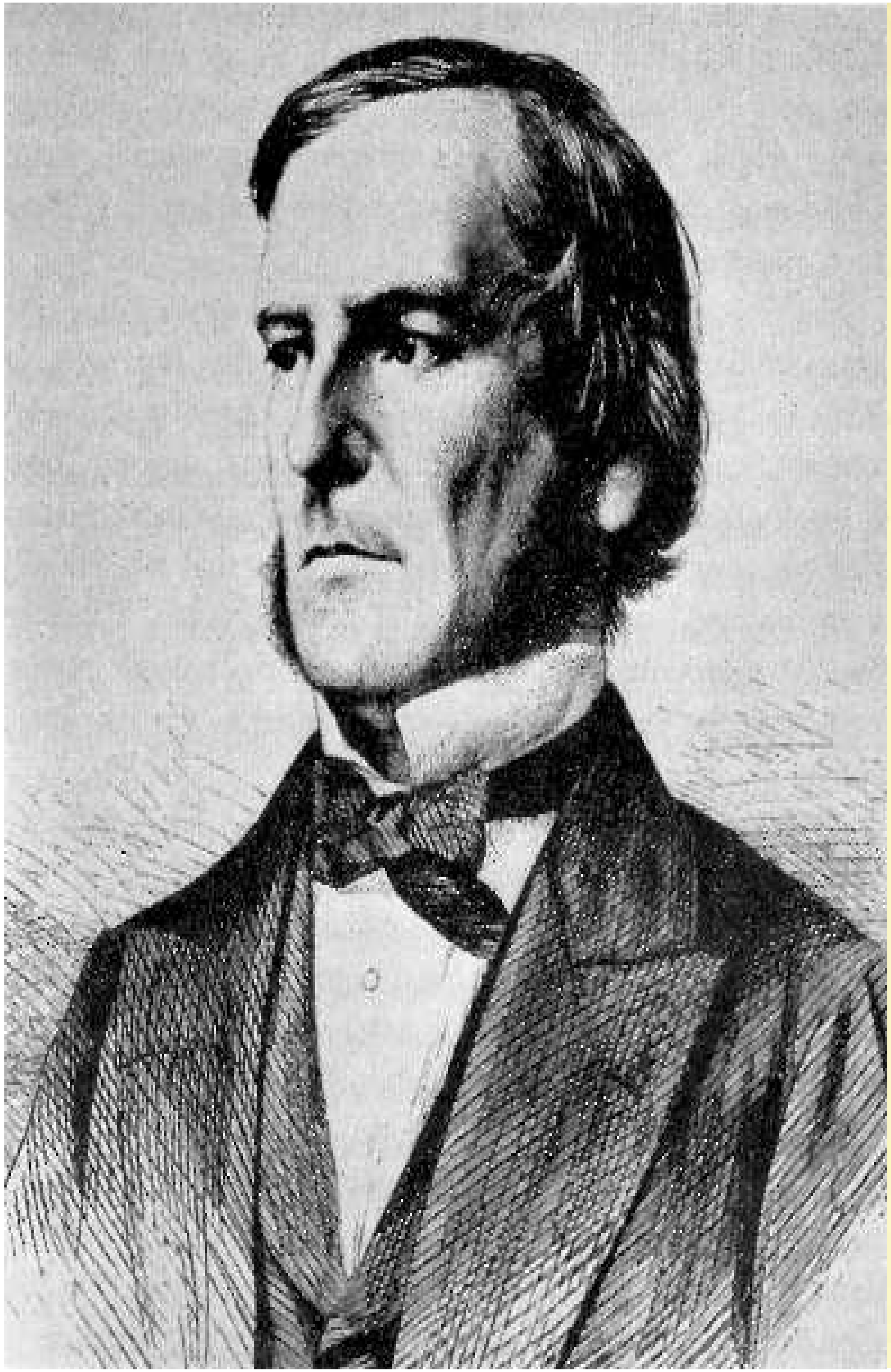
- ‘**the general problem of Formal Logic**’ (Boole, *On the Foundations of the Mathematical Theory of Logic*, 1856: 97)

- ‘George Boole […] first put forth **the problem of logical science in its complete generality**: — *Given certain logical premises or conditions, to determine the description of any class of objects under those conditions*. Such was the general problem of which the ancient logic had solved but a few isolated cases […] Boole showed incontestably that it was possible, by the aid of a system of mathematical signs, to deduce the conclusions of all these ancient modes of reasoning, and an indefinite number of other conclusions. Any conclusion, in short, that it was possible to deduce from any set of premises or conditions, however numerous and complicated, could be calculated by his method’ (W. S. Jevons, ‘On the mechanical performance of logical inference’, 1870: 499)

- ‘**the central problem of symbolic logic**’ (J. N. Keynes, *Formal Logic*, 1906: 506)



Natural Language
Concrete Premises
Elimination
Concrete Conclusion
Formalisation
Interpretation
Formal Premises
Calculus
Formal Conclusion
Formal Language



premises) with respect to the following elements, viz., the possession of red blood, of an external covering, and of a vertebral column.

We must first eliminate a. The result is

$$ir\,\{1 - sn\,(1-t) - st\,(1-n)\} + nt = 0.$$

Then (IX. 9) developing with respect to s and t, and reducing the first coefficient by Prop. 1, we have

$$nst + ir(1-n)s(1-t) + (ir+n)(1-s)t + ir(1-s)(1-t) = 0. \qquad (5)$$

Hence, if $st = w$, we find

$$nw + ir\,(1-n) \times (ir+n) \times ir\,(1-w) = 0\,;$$

or,

$$nw + ir\,(1-n)\,(1-w) = 0\,;$$

$$\therefore\ w = \frac{ir\,(1-n)}{ir\,(1-n) - n}$$

$$= 0\,irn + ir\,(1-n) + 0i\,(1-r)\,n + \frac{0}{0}\,i\,(1-r)\,(1-n)$$

$$+ 0\,(1-i)\,rn + \frac{0}{0}\,(1-i)\,r\,(1-n) + 0\,(1-i)\,(1-r)\,n$$

$$+ \frac{0}{0}\,(1-i)(1-r)(1-n)\,;$$

or,

$$w = ir\,(1-n) + \frac{0}{0}\,i\,(1-r)\,(1-n) + \frac{0}{0}\,(1-i)\,(1-n).$$

Hence, *soft-bodied animals enclosed in tubes consist of all invertebrate animals having red blood and not naked, and an indefinite remainder of invertebrate animals not having red blood and not naked, and of vertebrate animals which are not naked.*

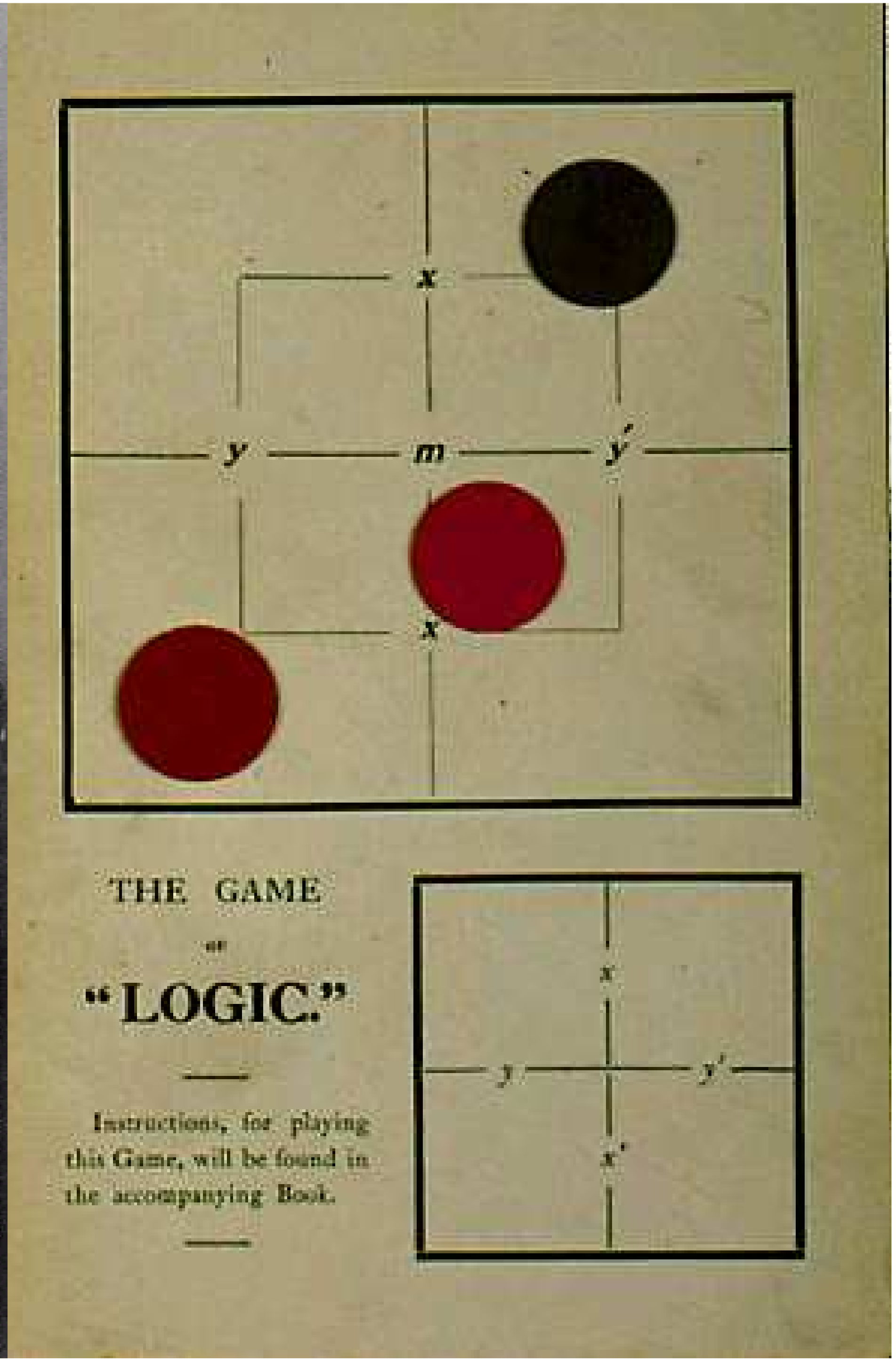
And in an exactly similar manner, the following reduced equations, the interpretation of which is left to the reader, have been deduced from the development (5).

$$s\,(1-t) = irn + \frac{0}{0}\,i\,(1-n) + \frac{0}{0}\,(1-i)$$

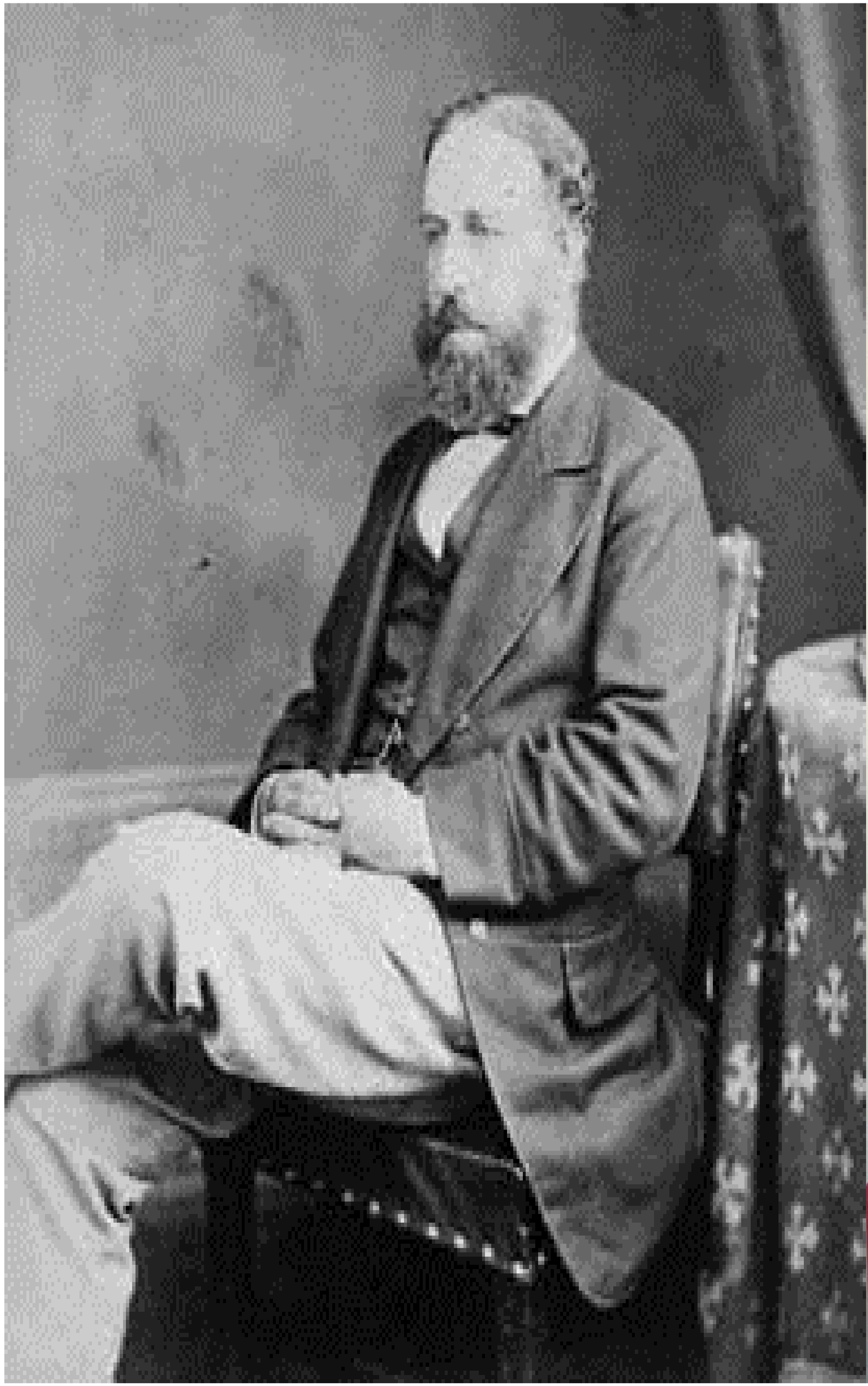
$$(1-s)\ t = \frac{0}{0}\,(1-i)\,r\,(1-n) + \frac{0}{0}\,(1-r)\,(1-n)$$

$$(1-s)\,(1-t) = \frac{0}{0}\,i\,(1-r) + \frac{0}{0}\,(1-i).$$

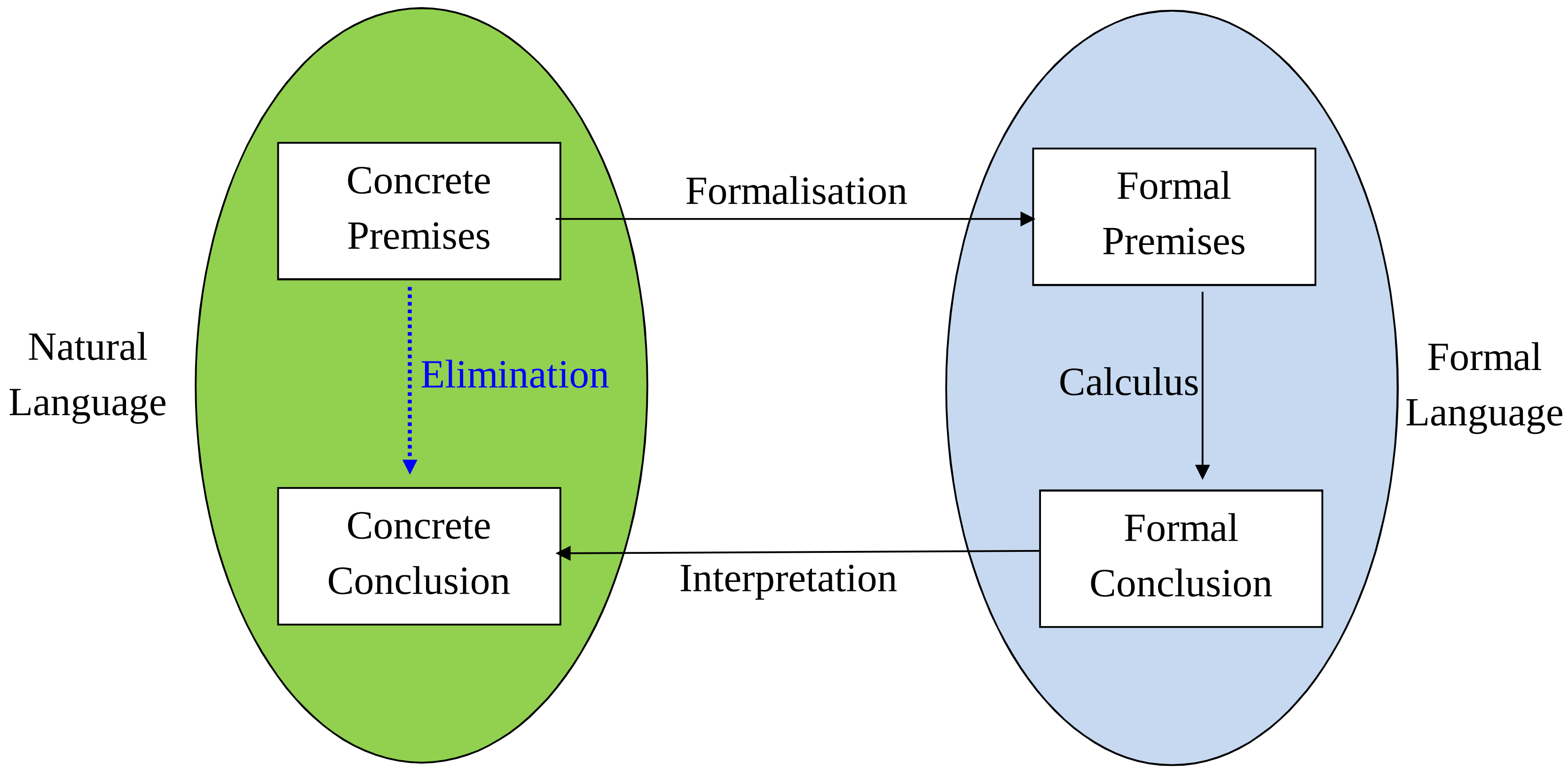
L



x
y
m
y'
x
THE GAME
of
"LOGIC."
Instructions, for playing this Game, will be found in the accompanying Book.
x
y
y'
x'



What (symbolic) notation?



Natural Language
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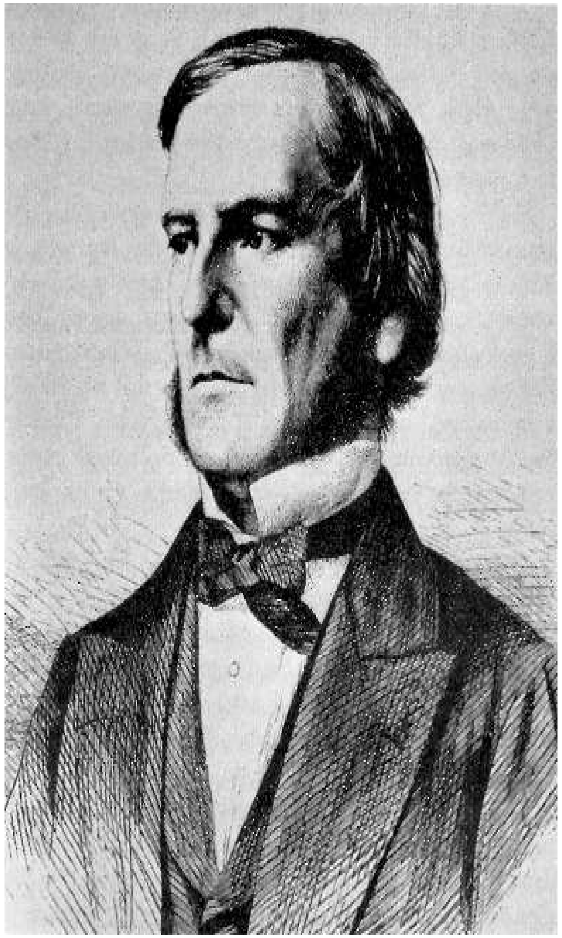
Formal Language



'Proposition I.

All the operations of Language, as an instrument of reasoning, may be conducted by a system of signs composed on the following elements, viz.:

1st. *Literal symbols, as x, y, & c., representing things as subjects of our conceptions.*

2nd. *Signs of operations, as* +, -, ×, *standing for those operations of the mind by which the conceptions of things are combined or resolved so as to form new conceptions involving the same elements.*

3rd. *The sign of identity,* =.

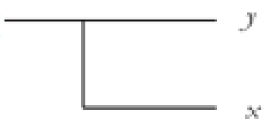
And these symbols of Logic are in their use subject to definite laws, partly agreeing with and partly differing from the laws of the corresponding symbols in the science of Algebra.'

(Boole, *LT,* 1854: 27)

'The great reform effected by Boole was that of making the equation the corner-stone of logic, as it had always been that of mathematical science. Not only did this yield true and simple results within the sphere of logic, but it disclosed wonderful analogy between logical and mathematical reforms'
(W. S. Jevons, 'Some recent mathematico-logical memoirs', 1881: 485-486)

1870s: inclusional notations

All x are y

- C. S. Peirce: $x \prec y$
- E. Schröder: $x \mathrel{\substack{<\\=}} y$
- H. MacColl: $x : y$
- L. Carroll: x y
- G. Frege: 

- *Convenience (MacColl)*

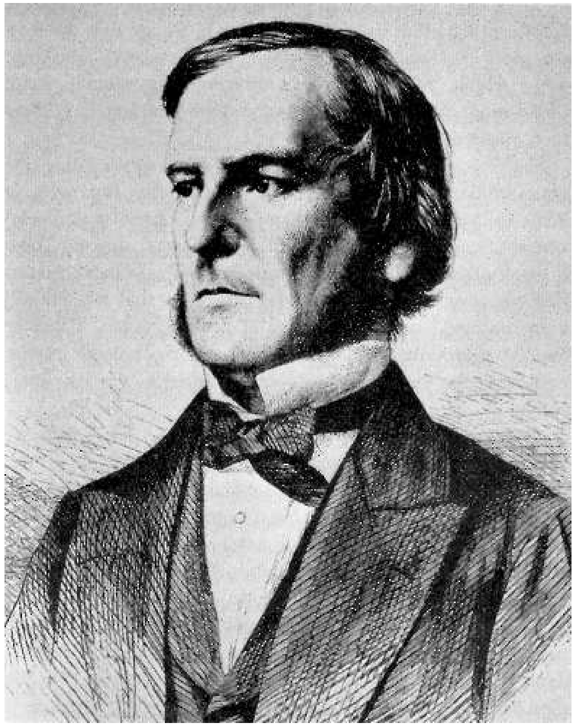
$(x : y)\ (y : z) : (x : z)$

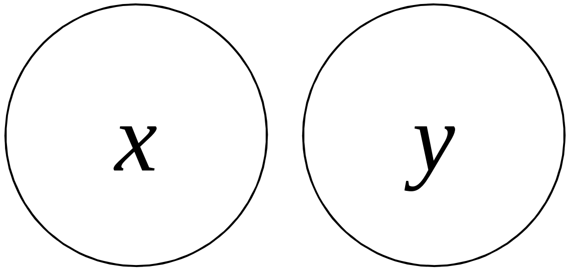
$(xy = x)\ (yz = y)\ (xz = x) = (xy = x)(yz = y)$

- *Simplicity (Peirce)*

$x \prec y$

$x = y$





No *x* is *y*

Boole

Peirce

$$x\,y = 0$$

Intersection of x and y is empty

$$x \prec \overline{y}$$

x is included in *not-y*

Ladd's exclusional notation

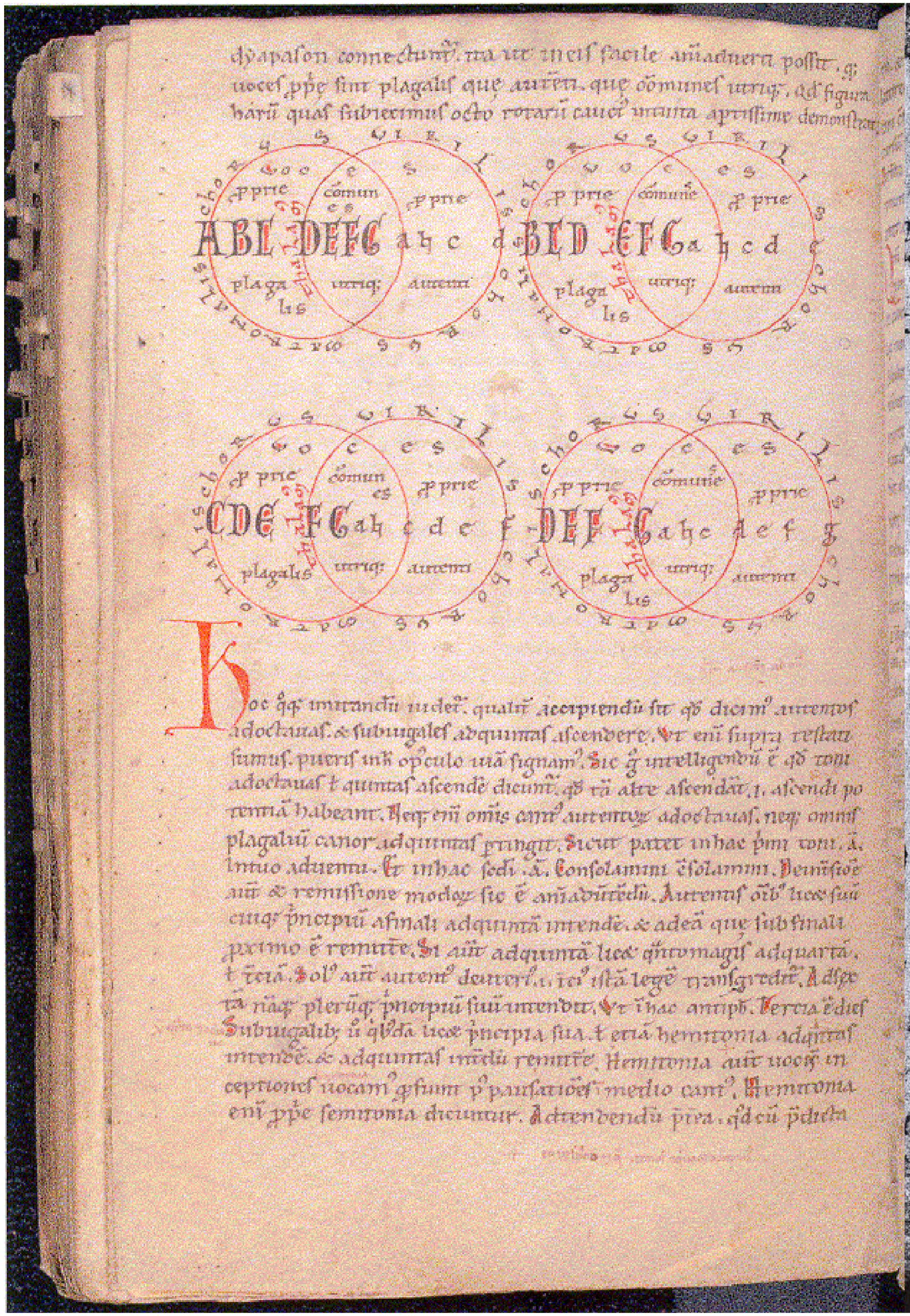
All x are y	$x \mathbin{-\!\!<} y$
Some x are y	$x \vee y$
No x is y	$x \; \overline{\vee} \; y$

Carroll's subscript notation

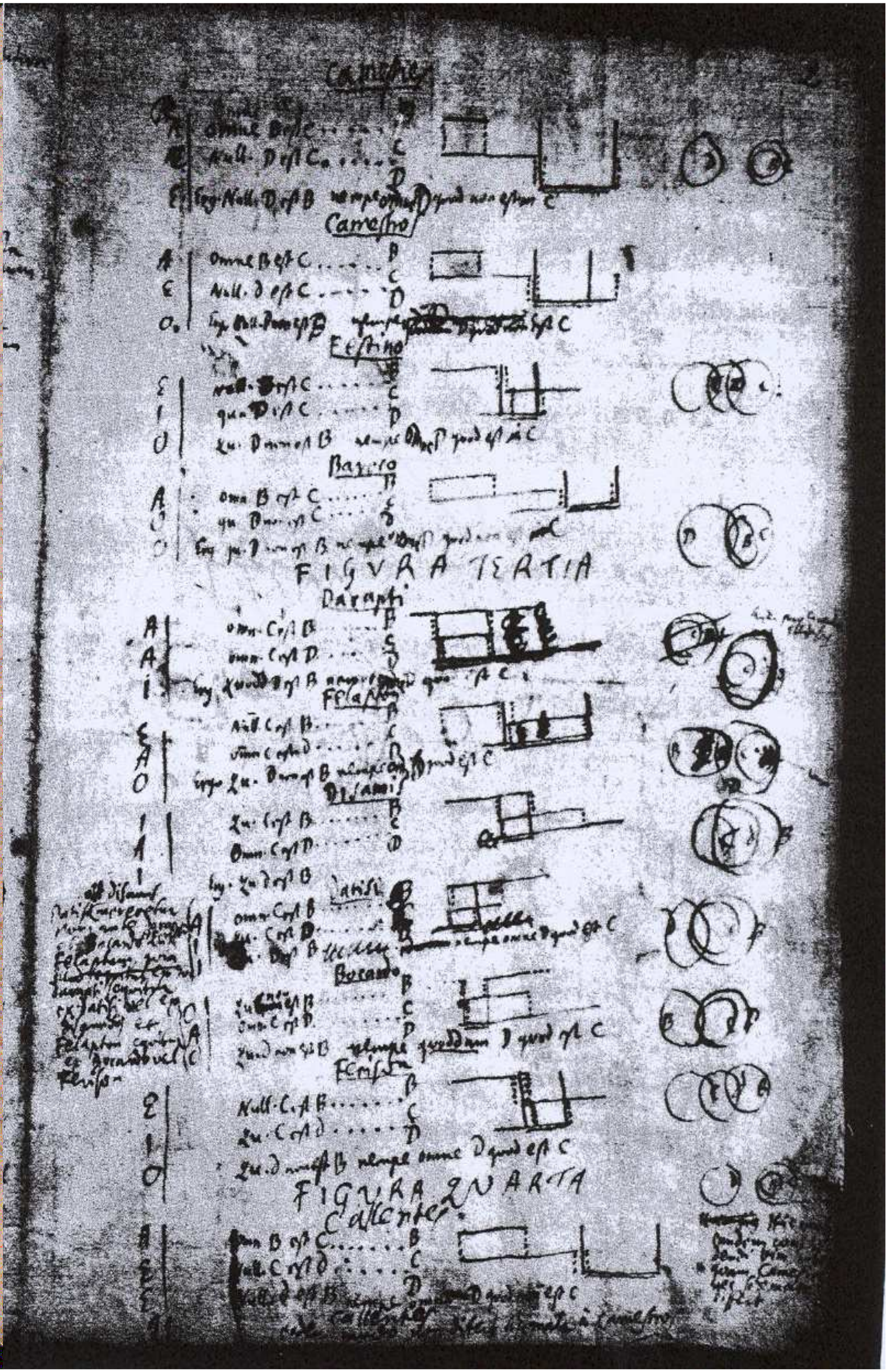
No *x* is *y* xy_0

Some *x* are *y* xy_1

What (diagrammatic) notation?



pprię
comun es
plagalis
utriq;
autenti



Camestres
Cesaro
Festino
Baroco
FIGVRA TERTIA
Darapti
Felapton
Disamis
Datisi
Bocardo
Ferison
FIGVRA QVARTA
Calentes

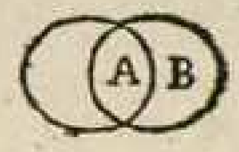
LETTRES A UNE PRINCESSE D'ALLEMAGNE

SUR DIVERS SUJETS de PHYSIQUE & de PHILOSOPHIE

TOME SECOND.

A SAINT PETERSBOURG
de l'Imprimerie de l'Academie Impériale des Sciences
M DCC LX VIII.

lieres, comme *quelque* A *eſt* B, une partie de l'eſpace A ſera compriſe dans l'eſpace B.



comme ici on voit viſiblement, que quelque choſe compriſe dans la notion A eſt auſſi compriſe dans la notion B.

IV. Pour les propoſitions négatives particulieres, comme *quelque* A *n'eſt pas* B; une partie de l'eſpace A doit ſe trouver hors de l'eſpace B: comme

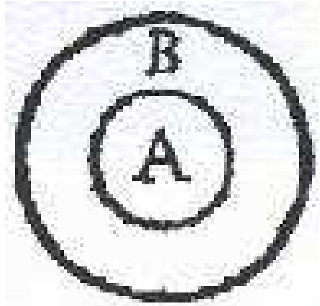
qui convient bien avec la précédente: mais on remarque ici principalement, qu'il y a quelque choſe dans la notion A, qui n'eſt pas compris dans la notion B, ou qui ſe trouve hors de cette notion.

le 14 Fevrier 1761.

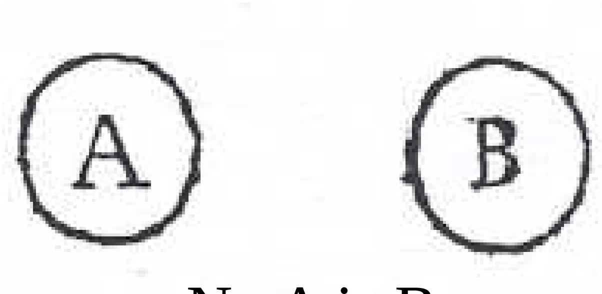
LETTRE CIII.

Ces figures rondes, ou plûtôt ces eſpaces, (car il n'importe quelle figure nous leur donnions) ſont très propres à faciliter nos réflexions ſur cette matiere, & à nous découvrir tous les

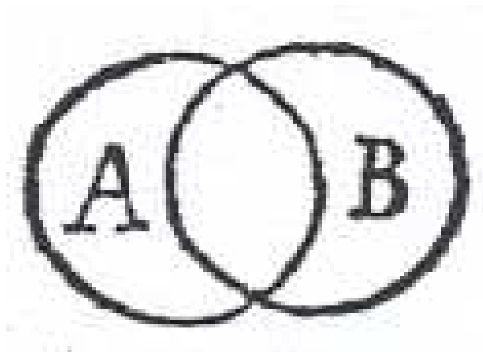
Euler diagrams



All A are B



No A is B

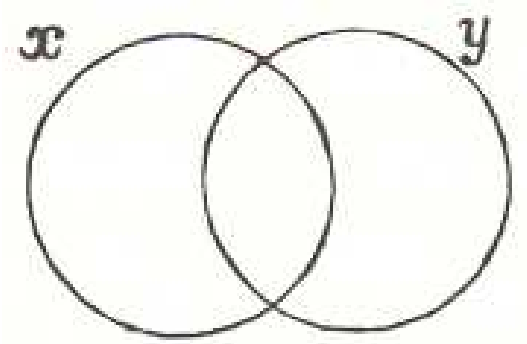


Some A are B

Venn diagrams

All *x* are *y*

$x (1-y) = 0$



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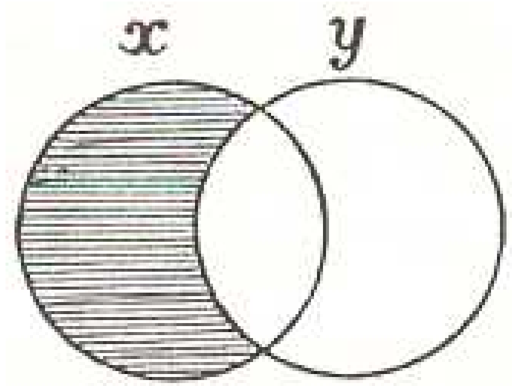
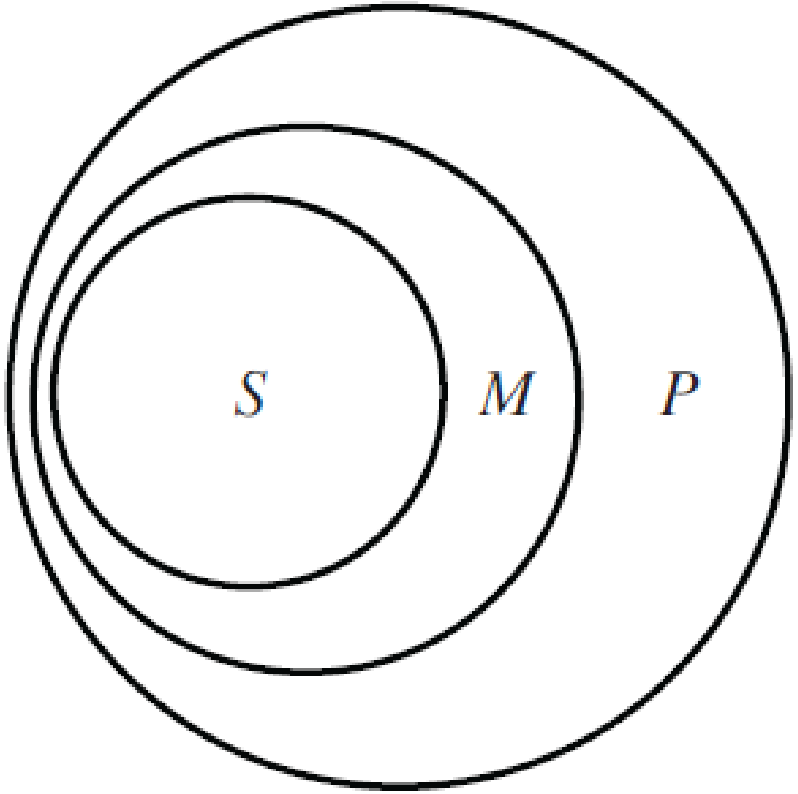
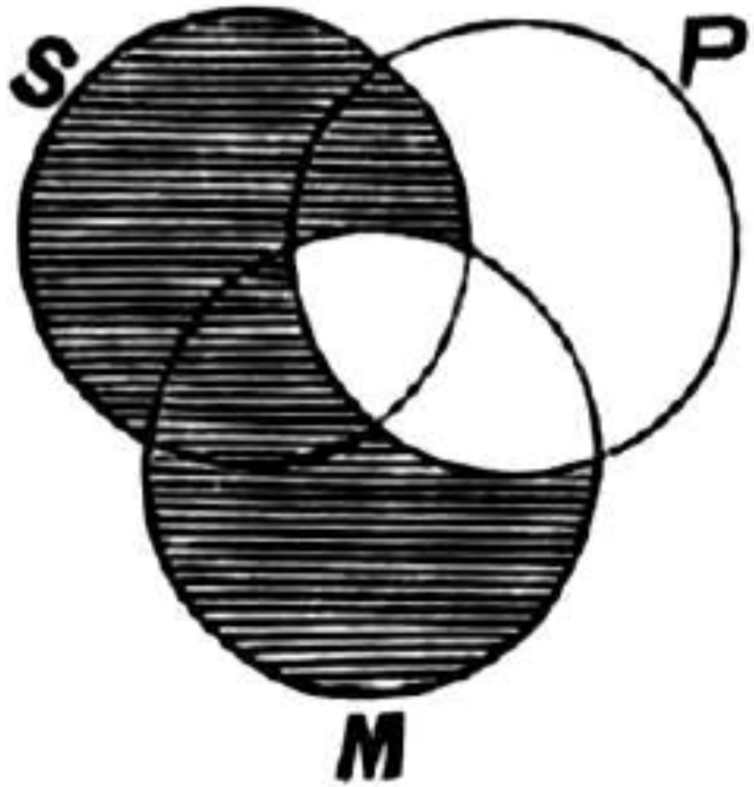
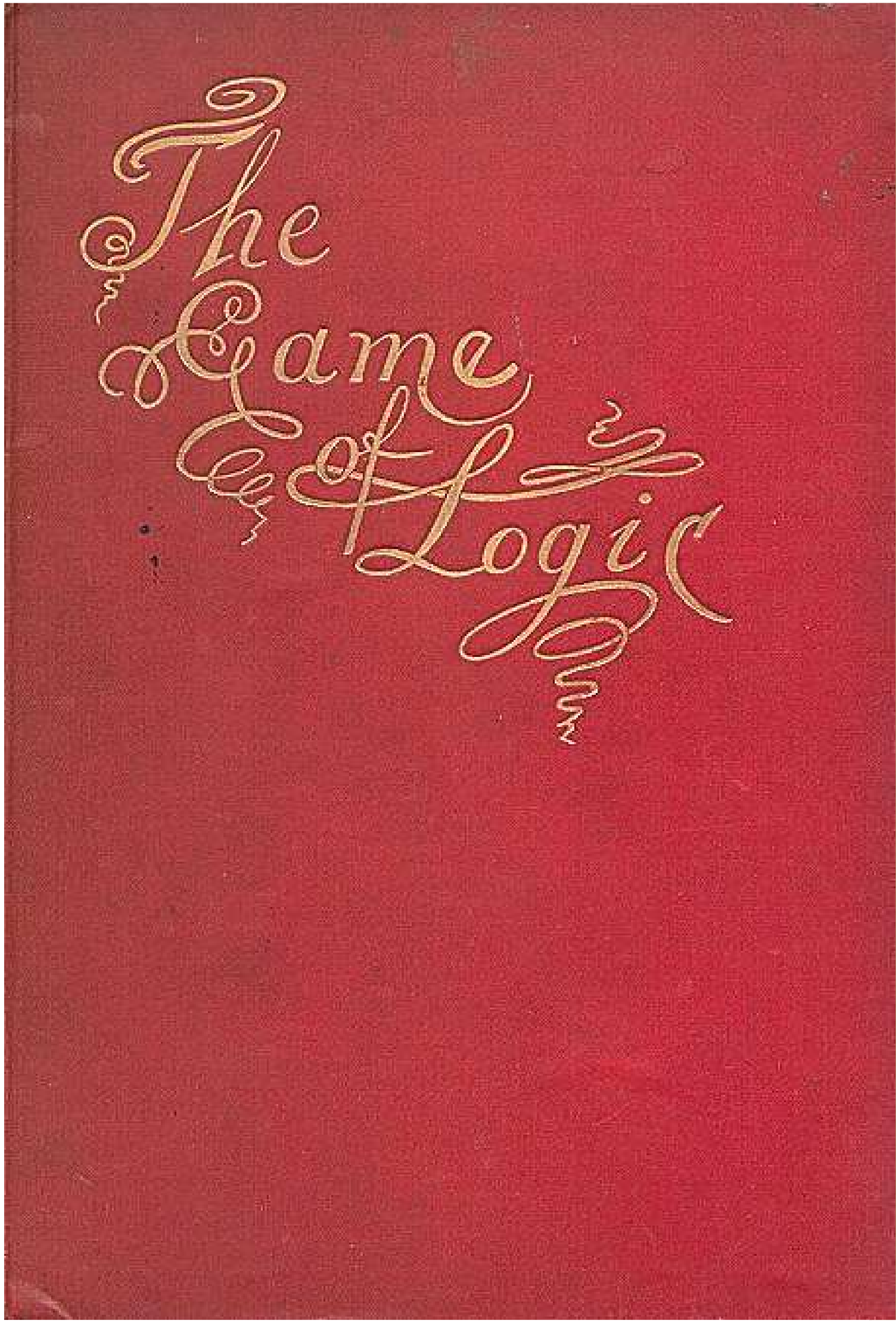
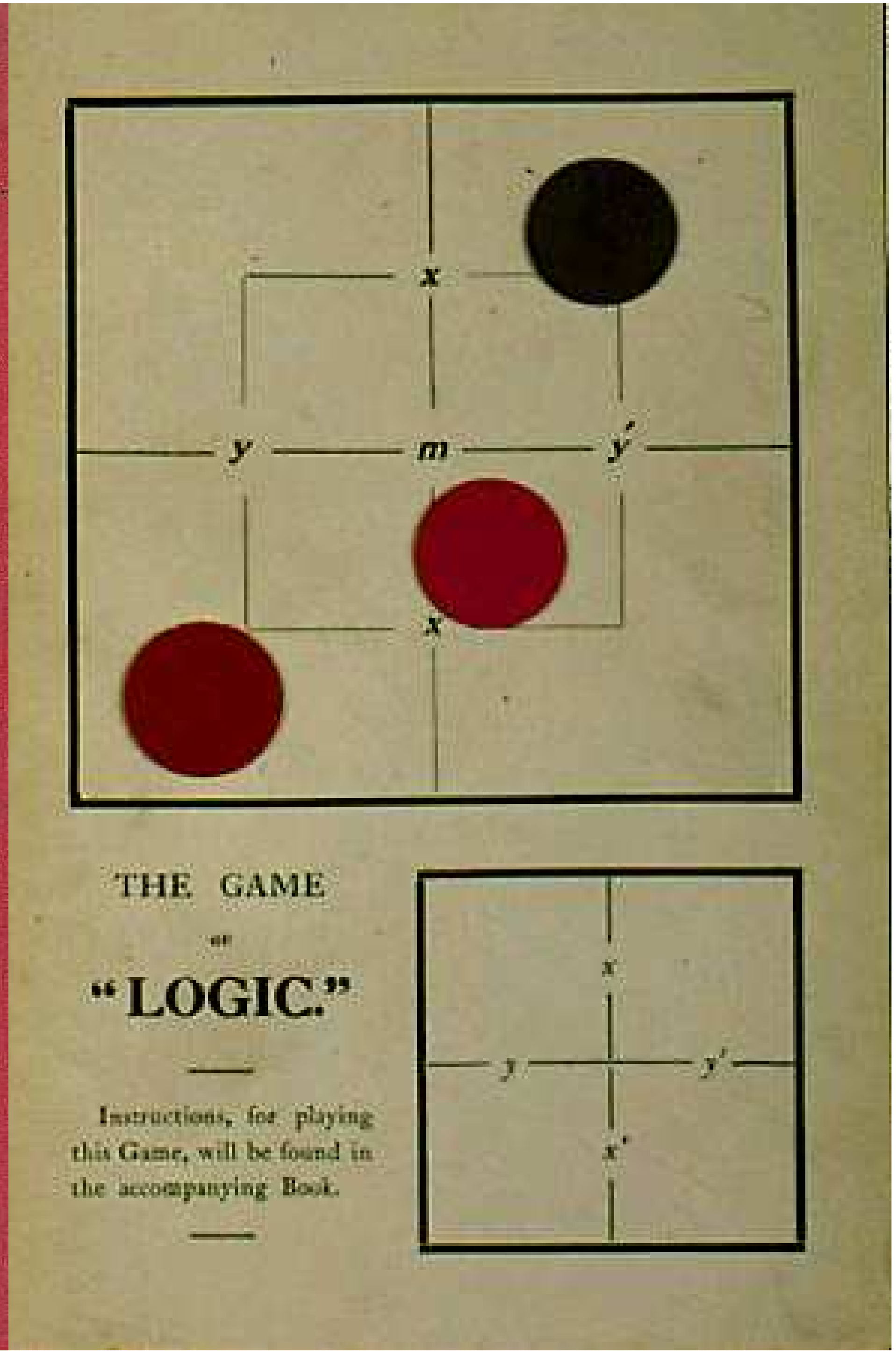



S
M
P


S
P
M


The Game of Logic

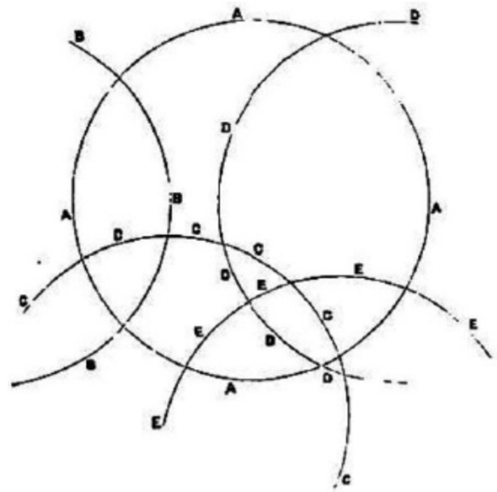

x
y
m
y′
x
THE GAME
of
"LOGIC."
Instructions, for playing this Game, will be found in the accompanying Book.
x
y
y′
x′

Diagrams for *n* terms (*n*>3)

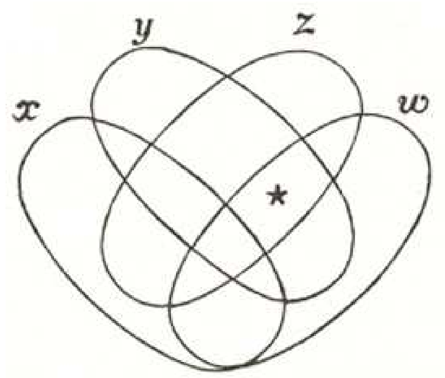
‘The traditional logic has been so entirely confined to the simultaneous treatment of three terms only (this being the number demanded for the syllogism) that hardly any attempts have been made to represent diagrammatically the combinations of four terms and upwards […]

Indeed, **except for those who wrote and thought under the influence of Boole**, directly or indirectly, it was scarcely likely that need should be felt for any more generalized scheme.’

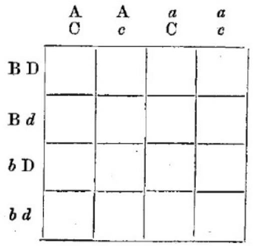
(J. Venn, *Symbolic Logic*, 1894: 511-513)



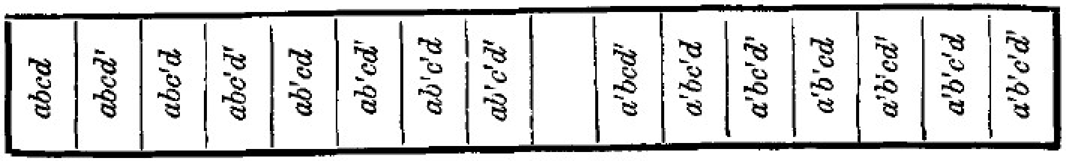
F. Garden, *Outline of Logic*, 1867: 39



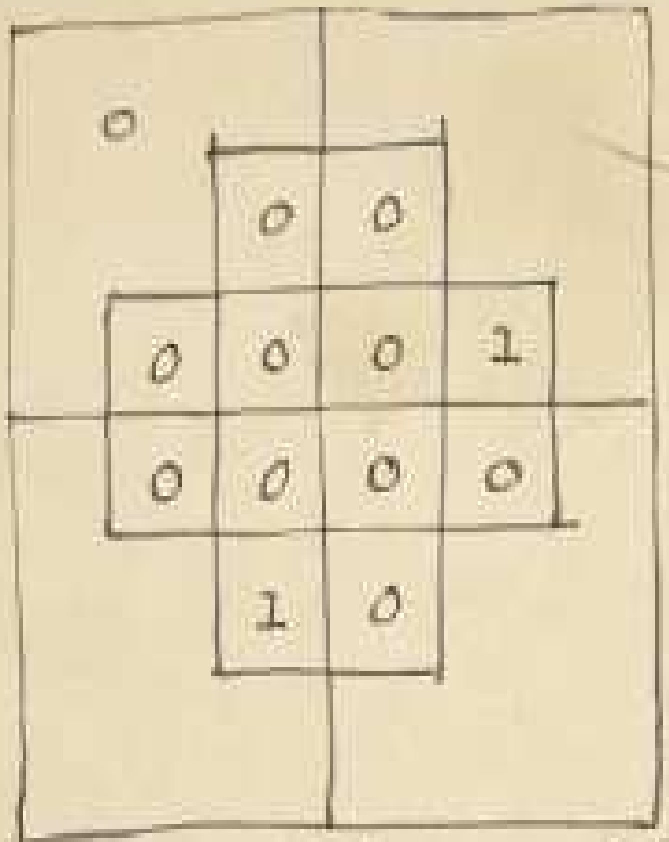
J. Venn, ‘On the diagrammatic and mechanical representation of propositions and reasonings’, 1880



A. Marquand, ‘Logical diagrams for n terms’, 1881

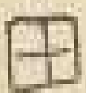


A. Macfarlane, ‘The logical spectrum’, 1885



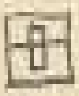
a = ducks
b = waltzers
c = officers
d = my poultry

ab_0
$c_1b'_0$
$d_1a'_0$



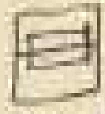
$\underline{ab}$

ab'_1
$a'b_1$



$\underline{ac}$

ac'_1
ac_0
$a'c_1$

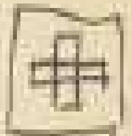


$\underline{ad}$

ad_1
$a'd'_1$

$\underline{bc}$

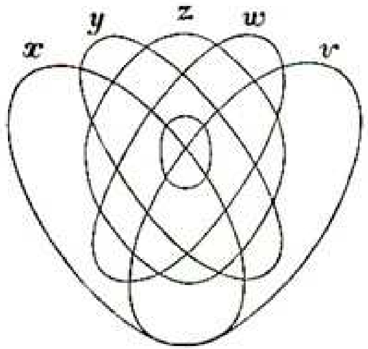
$\underline{bd}$

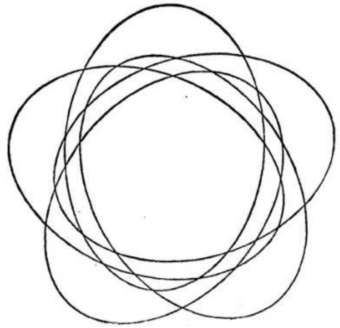


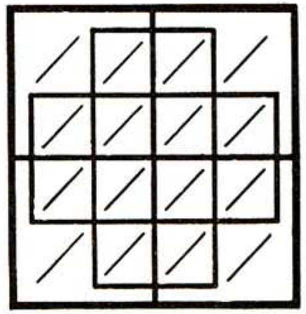
$\underline{cd}$

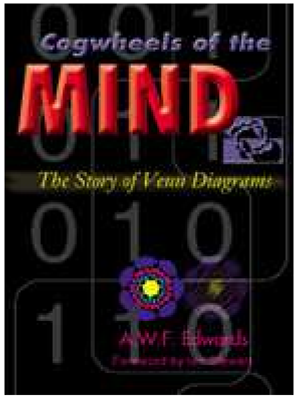
$\left.\begin{array}{l} cd_0 \\ c'd'_1 \end{array}\right\} c_1d_0$

$c'd_1$



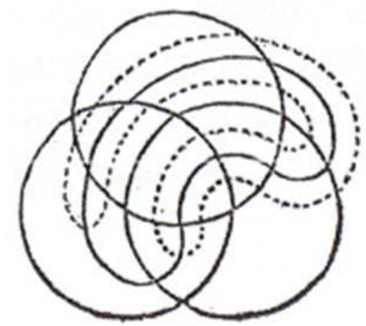


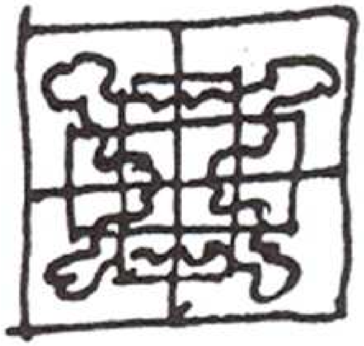




‘Both Venn and Carroll gave up at four sets and offered five-set diagrams whose fifth set did not consist of a closed curve, so that some regions became disjoint. In our terminology, they were not really Venn diagrams at all: once one admits the possibility of sets being bounded by more than one closed curve, one might as well just list all the binary numbers between 0 and 2^{n-1} and put a little ring round each!’

(A. W. F. Edwards, *The Story of Venn Diagrams*: 32)





A Friendly contest

MATHEMATICAL

QUESTIONS AND SOLUTIONS,

FROM THE "EDUCATIONAL TIMES,"

WITH MANY

PAPERS AND SOLUTIONS

IN ADDITION TO THOSE

PUBLISHED IN THE "EDUCATIONAL TIMES,"

WITH AN

APPENDIX.

EDITED BY

W. J. C. MILLER, B.A.,

REGISTRAR OF THE GENERAL MEDICAL COUNCIL.

VOL. LIII.

LONDON:

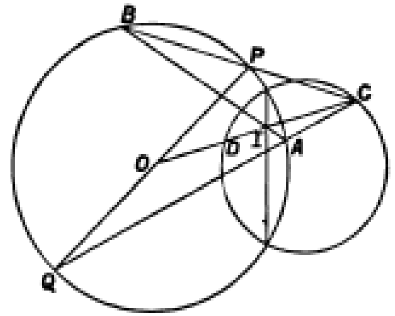
FRANCIS HODGSON, 89 FARRINGDON STREET, E.C.

1890.

mode of division chosen) the axial symmetry of the segments. To save space, in the subjoined examples, halves of mutually convertible figures are combined in the form of a single polygon. [We regret that we can afford space for only *one* of the very interesting illustrative diagrams sent by Mr. HERVEY.]

10402. (E. M. LANGLEY, M.A.) — If a diameter of one of two orthogonal circles cuts the common chord at I, and the other circle in C, and if AB is any chord of the first circle through I, show that CI is a symmedian of the triangle ABC.

Solution by Professor GENESE, M.A.



Let O be the centre of the first circle, OC meet the second circle at D, and CB, CA meet the first circle at P, Q; then

$$OD \,.\, OC = OP^2 = OD^2,$$

$\therefore$ $\angle OPD = OCP = OBD$,

hence O, D, P, B are concyclic, and $\angle POD = PBD$. So $\angle QOD = CAD$. Now

$$IA \,.\, IB = ID \,.\, IC;$$

hence A, B, C, D are concyclic, and angles CAD, CBD supplementary; $\therefore$ $\angle$'s POD, QOD are so, and P, O, Q are in a line. But PQ, AB are anti-parallel in angle ACB. Thus CI, a median to PQ, is a symmedian to AB.

10429. (A. E. JOLLIFFE.)—A parabola touches the sides of a triangle at A′, B′, C′; prove that AA′, BB′, CC′ meet on the minimum ellipse circumscribing the triangle.

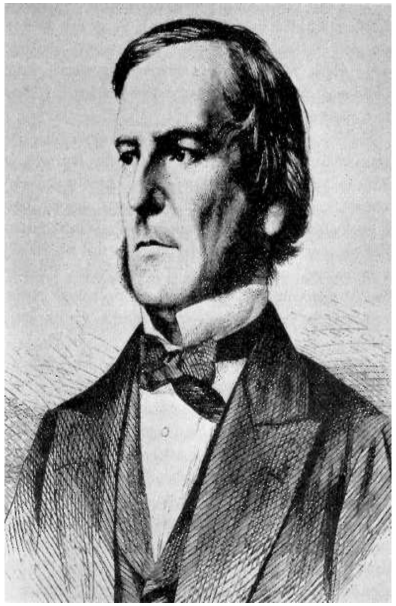
Solution by Hon. BERTRAND RUSSELL; Prof. ANDERSON; *and others.*

If the parabola be $\quad (lx)^{\frac12} + (my)^{\frac12} + (nz)^{\frac12} = 0,$

the intersection of AA′, BB′, CC′ is (l^{-1}, m^{-1}, n^{-1}). But $l+m+n=0$; hence the intersection lies on the conic $x^{-1}+y^{-1}+z^{-1}=0$, whose centre is $x=y=z$, *i.e.*, the centroid of the triangle. Hence the intersection lies on the minimum circumscribing ellipse.

[If we draw an equilateral triangle whose sides touch a given parabola, the lines that join any angular point with the point of contact of the opposite side will each pass through the focus, which is a point on the circumcircle of the triangle. Projecting orthogonally, &c.]

Boole's 5th problem



'Ex. 5. Let the observation of a class of natural productions be supposed to have led to the following general results.

1st, That in whichsoever of these productions the properties A and C are missing, the property E is found, together with one of the properties B and D, but not with both.

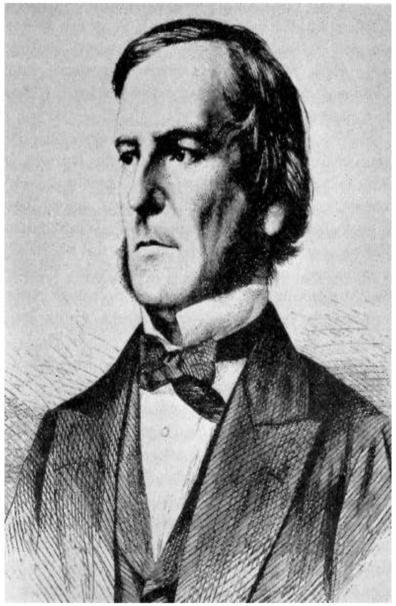
2nd, That wherever the properties A and D are found while E is missing, the properties B and C will either both be found or both be missing.

3rd, That wherever the property A is found in conjunction with either B or E, or both of them, there either the property C or the property D will be found, but not both of them. And conversely, wherever the property C or D is found singly, there the property A will be found in conjunction with either B or E, or both of them.

Let it then be required to ascertain, first, what in any particular instance may be concluded from the ascertained presence of the property A, with reference to the properties B, C, and D; also whether any relations exist independently among the properties B, C, and D. Secondly, what may be concluded in like manner respecting the property B, and the properties A, C, and D'

(Boole, *LT*, 1854: 146)

Boole's 5th problem



'In none of the above examples has it been my object to exhibit in any special manner the power of the method [...] I would, however, suggest to any who may be desirous of forming a correct opinion upon this point, that they examine by the rules of ordinary logic the following problem, *before* inspecting its solution; remembering at the same time that, that whatever complexity it possesses might be multiplied indefinitely, with no other effect than to render its solution by the method of this work more operose, but not less certainly attainable.'
(Boole, *LT*, 1854: 146)

I have not yet attempted to extend the inquiry to frames of 7 or more joints. There are even cases where a frame of *less* than the number $2n-3$ of bars required for stiffness may be self-strained. Thus, a frame of the form of a regular octagon, with four diagonal bars crossing at the centre, would evidently be in equilibrium, if the diagonals be all in tension, of the same amount, and the sides in compression of the amount requisite to give equilibrium at each joint: yet the frame is not even rigid, and would require a thirteenth bar to render it so.*

The Calculus of Equivalent Statements (*Third Paper*).

By Hugh McColl, B.A.

[*Read November 14th*, 1878.]

The following formulæ are all either self-evident or easily verified, and some of them will be found useful in abbreviating the operations of the calculus:—

(1) $1'=0, \quad 0'=1$;

(2) $1=1+a=1+a+b=1+a+b+c$, &c.;

(3) $(ab+a'b')'=a'b+ab'$,
$(a'b+ab')'=ab+a'b'$;

(4) $a : a+b : a+b+c$, &c.;

(5) $(a+A)(a+B)(a+C)\ldots=a+ABC\ldots$;

(6) $(a : b) : a'+b$;

(7) $(a=b)=(a : b)(b : a)$;

(8) $(a=b) : ab+a'b'$;

(9) $(A : a)(B : b)(C : c)\ldots : (ABC\ldots : abc\ldots)$;

(10) $(A : a)(B : b)(C : c)\ldots : (A+B+C+\ldots : a+b+c+\ldots)$;

(11) $(A : x)(B : x)(C : x)\ldots=(A+B+C+\ldots : x)$;

(12) $(x : A)(x : B)(x : C)\ldots=(x : ABC\ldots)$;

(13) $(A : x)+(B : x)+(C : x)+\ldots : (ABC\ldots : x)$;

(14) $(x : A)+(x : B)+(x : C)+\ldots : (x : A+B+C+\ldots)$.

Rule 19.—To test the equivalence of any two statements; say

$$f(a, b, c, \ldots) \text{ and } \phi(a, b, c, \ldots).$$

* It is easy to show that the only condition necessary for any octagonal frame, with 4 diagonal bars joining opposite vertices, to be capable of internal stress, is that the 4 intersections of each pair of opposite sides shall be *in directum*.

Hence, adding antecedent to antecedent, and consequent to consequent (see Formula 10), we get (since $E+E'=1$)

$$C : AB+A'B'+A'D' \ldots\ldots\ldots\ldots (1),$$

omitting two redundant terms ABD and $A'B'$.

Again, from the given premises we get

$$CE : (A=0) : A',$$
$$C'E' : (A=0) : A'.$$

Hence $$C' : A' \ldots\ldots\ldots\ldots (2).$$

From (1) and (2) we get, by transposition,

$$\overline{AB'}+A'BD : C',$$
$$A : C.$$

The term AB' over-lined in the first of these two implications may be cancelled as zero,* since it is a multiple of the term A in the second implication (see Rule 24). Cancelling this term and transposing again, we get, for our final result,

$$A : C : A+B'+D'.$$

Another problem from Boole (see "Laws of Thought," page 146), when translated into the language of this calculus, is the following:—

From the premises expressed by the complex statement

$$\{A'C' : E(B'D+BD')\}(ADE' : BC+B'C')\{A(B+E)=C'D+CD'\},$$

it is required, (1) to express A in terms of B, C, D, eliminating E; and (2) to express B in terms of A, C, D.

Using Rule 23 and Formulæ 6 and 8 of this paper, we get

$$AE : C'D+CD',$$
$$AE' : (D'+BC+B'C')\{B(C'D+CD')+B'(CD+C'D')\}.$$

Adding antecedent to antecedent, and consequent to consequent (see Formula 10), and omitting the term $B(C'D+CD')$ in the double bracket, because it is a multiple of the first consequent $C'D+CD'$, we

get $$A : C'D+CD'+B'(D'+\overline{BC}+B'C')(CD+C'D').$$

We may cancel the term over-lined because it is inconsistent with the outside factor B'.

Hence $$A : C'D+CD'+B'(C'D'+\underline{B'C'D'}),$$

that is, $$A : C'D+CD'+B'C'D'.$$

Hence, by transposition, $CD+BC'D' : A'$.

* From this zero term we may deduce the two implications $A : B$ and $B' : A'$.

SYMBOLIC LOGIC

BY

JOHN VENN, M.A.,

FELLOW, AND LECTURER IN THE MORAL SCIENCES,

GONVILLE AND CAIUS COLLEGE, CAMBRIDGE.

"Sunt qui mathematicum vigorem extra ipsas scientias, quas vulgo mathematicas appellamus, locum habere non putant. Sed illi ignorant, idem esse mathematice scribere quod in forma, ut logici vocant, ratiocinari."

LEIBNITZ, *De vera methodo Philosophiae et Theologiae* (about 1690).

"Cave ne tibi imponant mathematici logici, qui splendidas suas figuras et algebraicos maeandros universale inventionis veri medium crepant."

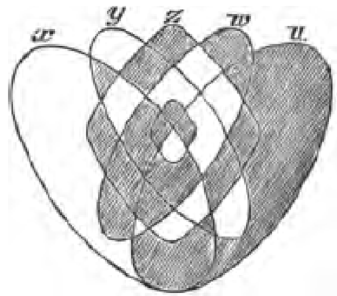
RÜDIGER, *De sensu veri et falsi*, Lib. II. Cap. IV. § xi. (1722).

London:

MACMILLAN AND CO.

1881

Now draw the 5-term diagram, and shade out the terms thus marked[1]. (On the pieced diagram board, described in Chap. V., they could be picked out and removed in a few minutes.) We have the following result:—



On looking at this diagram, several of the various conclusions which Boole has drawn are almost intuitively obvious. Thus that 'there is no xzw' ($xzw = 0$); that 'all w is either x or z' ($\bar{x}\bar{z}w = 0$); that 'all z is either x or w' ($\bar{x}z\bar{w} = 0$). These are the sort of conclusions to which diagrams specially lend themselves; for in each case we extinguish a connected group of classes, and each extinction readily catches the eye in a figure.

Similarly it is not difficult to verify the conclusion that 'wherever x is found there will be found either z or w (but not both) or else y, z, and w will all be absent; and conversely' (Boole, p. 148). Symbolically this stands

$$x = z\bar{w} + \bar{z}w + \bar{y}\bar{z}\bar{w}.$$

On looking at the composition of x in the diagram it will readily be seen that it is made up of these three (in their ultimate subdivision, *six*) constituents. This sort of conclusion, though easy to verify by a figure, is probably easier to obtain (apart from the extreme and inevitable tediousness) from the symbolic letters.

[1] We have not troubled to shade the outside of this diagram, viz. $\bar{x}\bar{y}\bar{z}\bar{w}\bar{u}$.

VORLESUNGEN

ÜBER DIE

ALGEBRA DER LOGIK

6.15.05

(EXAKTE LOGIK)

VON

DR. ERNST SCHRÖDER,

ORD. PROFESSOR DER MATHEMATIK AN DER TECHNISCHEN HOCHSCHULE ZU KARLSRUHE IN BADEN,
KORRESPONDIRENDEM MITGLIEDE DER BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

ERSTER BAND.

MIT VIEL FIGUREN IM TEXTE.

Der Mensch ist nicht geboren, das Problem der Welt zu lösen, wohl aber, zu suchen, wo das Problem angeht, und sich sodann in den Grenzen des Begreiflichen zu halten.
Goethe, Eckermann's Gespräche; Okt. 1825.

Ich sag' es dir: ein Kerl, der spekulirt,
Ist wie ein Tier, auf dürrer Haide
Von einem bösen Geist im Kreis herumgeführt,
Und rings umher liegt schöne grüne Weide.
Derselbe (Mephisto).

LEIPZIG,
DRUCK UND VERLAG VON B. G. TEUBNER.
1890.

legungsweisen in's rechte Licht zu setzen, jene als die überlegene zu erproben.

Dagegen wolle man diesen Beispielen nicht etwa die Bestimmung zuschreiben, dass sie den *Nutzen* unsrer Kunstlehre des Denkens — vielleicht für das praktische Leben — darzuthun hätten.*) Utilitarische Bestrebungen liegen uns nach wie vor ferne und setzen wir voraus, dass auch der Leser von dem wissenschaftlichen Interesse geleitet sei.

Ich gebe die Aufgaben nicht etwa peinlich nach ihrer Schwierigkeit geordnet. Der Studirende, welcher mit den leichtesten beginnen und von diesen allmälig aufsteigend zu den verwickelteren fortschreiten will („schwierige" gibt es eigentlich unter den bisherigem Kalkul überhaupt zugänglichen Problemen, nachdem derselbe so weit entwickelt ist, nicht mehr) braucht sich nur zuerst an diejenigen zu machen, welchen der geringste Druckumfang gewidmet ist, und bei denen sich am wenigsten Formelanhäufungen dem Auge darbieten!

Ich beginne vielmehr mit jener komplizirtesten der von Boole gestellten Aufgaben, welche ich erstmalig in[2] nach seiner geläuterten Methode behandelt habe und auch hier mit allen Zwischenrechnungen durchnehme — weil mir dieselbe jenen oben angedeuteten Zwecken der Methodenerläuterung und später auch -vergleichung am vielseitigsten und besten zu dienen fähig erscheint.

1. Aufgabe. (Boole[4] p. 146 .. 149.) Es werde (gemäss Boole) angenommen**), dass die Beobachtung einer Klasse von Erscheinungen (Natur- oder Kunsterzeugnissen, z. B. Substanzen) zu den folgenden allgemeinen Ergebnissen geführt hat:

α) Dass in welchem auch von diesen Erzeugnissen die Merkmale A und C gleichzeitig fehlen, das Merkmal E gefunden wird, zusammen mit einem der beiden Merkmale B und D, aber nicht mit beiden.

β) Dass, wo immer die Merkmale A und D in Abwesenheit von E gleichzeitig auftreten, die Merkmale B und C entweder beide sich vorfinden oder beide fehlen.

γ) Dass überall, wo das Merkmal A mit dem B oder E, oder mit beiden zusammen besteht, auch entweder das Merkmal C vorkommt oder das D, aber nicht beide. Und umgekehrt, überall wo von den Merkmalen C und D das eine ohne das andre wahrgenommen wird, da soll auch

*) Dafür sind sie meistens zu künstlich ersonnen. Zum Teil werden die Aufgaben *mehr* nur mit Scherzrätseln, Vexiraufgaben, Spielproblemen verwandt erscheinen.

**) Über die Zulässigkeit (in gewissem Sinne Unzulässigkeit) dieser Annahme vergleiche die unten folgende „Anmerkung" zur Aufgabe.

In testing after obtaining B and four sums, the work can be used as a matrix for making 5 separate columns, when another figure will be unchangeable and C will be increased 25 times. After obtaining ten sums, in each of these five columns, they may be used as matrices, for forming ten columns, and C will be increased to 450 billions, at least.

In operating, the several columns should be carried along together, so that nearly equal sums may be obtained successively.

In protracted search, I propose to work downward from the upper limits, by using several principles, obtained from equations involving $\frac{1}{4}(N \pm 1)$, and also to test the sums obtained where the table of squares is not sufficiently extended. Also I propose a system of abridged multiplication for finding the unchangeable part of roots, where a table of squares is not available.

APPLICATION OF THE METHOD OF THE LOGICAL SPECTRUM TO BOOLE'S PROBLEM. By A. MACFARLANE, Prof. of Physics, University of Texas, Austin, Texas.

PROFESSOR SCHROEDER of Karlsruhe, by the publication of his "Verlesungen über die Algebra der Logic," has brought this branch of mathematics more generally to the notice of mathematicians. One of the problems he discusses is one which Boole proposed and solved as a specimen of the power of his method (Laws of Thought, p. 146). Boole says at the end of his investigation that he has not attempted to verify his conclusions. I propose to solve and verify by the method of the logical spectrum. The problem is:—

"Let the observation of a class of natural productions be supposed to have led to the following general results:—

(1) That in whichsoever of these productions the properties A and C are missing, the property E is found, together with one of the properties B and D, but not with both.

(2) That wherever the properties A and D are found while E is missing, the properties B and C will either both be found or both be missing.

(3) That wherever the property A is found in conjunction with either B or E, or both of them, there either the property C or the property D will be found, but not both of them. And, conversely, wherever the property C or D is found singly, there the property A will be found in conjunction with either B or E, or both of them.

Let it then be required to ascertain, first, what in any particular instance may be concluded from the ascertained presence of the property A, with reference to the properties B, C, D; also whether any relations exist independently among the properties B, C, D. Secondly, what may be concluded in like manner respecting the property B, and the properties A, C, D."

all the equations are indefinite, then a contains an indefinite portion of that class. If the only definite value for a is 1, then a contains the whole of that class; if 0, then none. If one equation gives $a=1$ and another $a=0$, then that class must be impossible.

By this method we get for a:—

$bcde$	none	$b'cde$	none
$bcde'$	none	$b'cde'$	none
$bcd'e$	all	$b'cd'e$	all
$bcd'e'$	all	$b'cd'e'$	impossible
$bc'de$	all	$b'c'de$	all
$bc'de'$	impossible	$b'c'de'$	impossible
$bc'd'e$	none	$b'c'd'e$	impossible
$bc'd'e'$	impossible	$b'c'd'e'$	all

Hence $a = bcd'e + bcd'e' + bc'de + b'cd'e + b'c'de + b'c'd'e'$, and by taking into account the impossible terms e and e' may be eliminated by addition, thus $a = cd' + c'd + b'c'd'$.

If we consider the elimination of e from the impossible terms, we find that the result is entirely indefinite. This is the answer to the first question.

To answer the second question, we form the sub-classes, due to the presence or absence of a, c, d, e, and by solving for a we obtain:

$acde$	impossible	$a'cde$	indefinite
$acde'$	impossible	$a'cde'$	indefinite
$acd'e$	indefinite	$a'cd'e$	impossible
$acd'e'$	all	$a'cd'e'$	impossible
$ac'de$	indefinite	$a'c'de$	impossible
$ac'de'$	impossible	$a'c'de'$	impossible
$ac'd'e$	impossible	$a'c'd'e$	all
$ac'd'e'$	none	$a'c'd'e'$	impossible

Hence, $b = a'c'd' +$ an indefinite portion of $(acd' + ac'd + a'cd)$ and there is a relation independent of b, namely:

$$acd + a'cd' + a'c'd = 0$$

These are the analytical expressions for Boole's second answer.

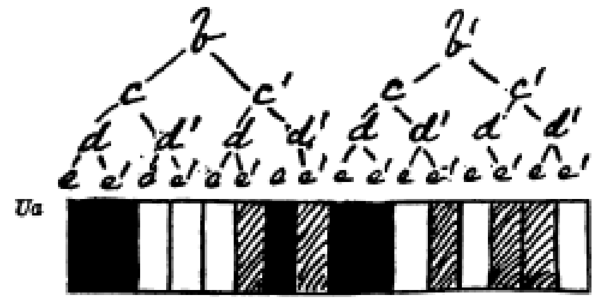


DIAGRAM 1.

The diagrammatic verification is as follows:—Let a strip represent U, the whole collection of productions. In diagram (1) Ua is represented by

Peirce and his ‘school’

As an example of this method, we may consider a well-known problem given by Boole. The data are

$$\bar{x} \times \bar{z} \prec v \times (y \times \bar{w} + \bar{y} \times w)$$
$$\bar{v} \times x \times w \prec (y \times z) + (\bar{y} \times \bar{z})$$
$$(x \times y) + (v \times x \times \bar{y}) = (z \times \bar{w}) + (\bar{z} \times w).$$

C. S. Peirce, ‘On the algebra of logic’, 1880: 39

STUDIES IN LOGIC.

BY MEMBERS

OF THE

JOHNS HOPKINS UNIVERSITY.

[Ed. Charles Santiago Sanders Peirce]

BOSTON:
LITTLE, BROWN, AND COMPANY.
1883.

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ON THE ALGEBRA OF LOGIC.

BY CHRISTINE LADD.

THERE are in existence five algebras of logic,—those of Boole, Jevons, Schröder, McColl, and Peirce,—of which the later ones are all modifications, more or less slight, of that of Boole. I propose to add one more to the number. It will bear more resemblance to that of Schröder than to any of the others; but it will differ from that in making use of a copula, and also in the form of expressing the conclusion.[1]

ON IDENTICAL PROPOSITIONS.

The propositions which logic considers are of two kinds,—those which affirm the identity of subject and predicate, and those which do not. Algebras of logic may be classified according to the way in which they express propositions that are not identities. Identical propositions have the same expression in all. Of the logical theorems which are identities, I shall give those which are essential to the subject, and for the most part without proof.

(1) The sign $=$ is the sign of equality. $a = b$, a equals b, means that in any logical expression a can

[1] The substance of this paper was read at a meeting of the Metaphysical Club of the Johns Hopkins University, held in January, 1881.

premises. As Mr. Jevons himself says: "It is hardly possible to apply this process to problems of more than six terms, owing to the large number of combinations which would require examination" (XIII., p. 96).

7. (III., p. 146). From the premises

$$\bar x\bar z(\bar v + wy + \bar w\bar y)\overline{\vee}$$
$$\bar v xw(y\bar z + \bar y z)\overline{\vee}$$
$$x(v + y)(zw + \bar z\bar w)\overline{\vee}$$
$$(\bar x + \bar v\bar y)(z\bar w + \bar z w)\overline{\vee}$$

it is required, first, to eliminate v; second, to express the conclusion in terms of x; third, in terms of y; fourth, to eliminate x; fifth, to eliminate y.

The terms which involve v are

$$\bar x\bar z + xw(y\bar z + \bar y z) + \bar y(z\bar w + \bar z w)\overline{\vee}\,\bar v,\quad x(zw + \bar z\bar w)\overline{\vee}\,v;$$

whence, taking the product of the left-hand members, we have only

$$xz\bar y w\,\overline{\vee}, \qquad (a)$$

which is to be added to that part of the premises which does not contain v,—namely, to

$$\bar x\bar z(wy + \bar w\bar y) + xy(zw + \bar z\bar w) + \bar x(z\bar w + \bar z w)\overline{\vee}.$$

Collecting the parts which contain x and $\bar x$ we have

$$x\,\overline{\vee}\,zw + y\bar z\bar w, \qquad (b)$$
$$\bar x\,\overline{\vee}\,z\bar w + \bar z w + \bar z\bar w\bar y. \qquad (c)$$

The negative of the second member of (c) is, by (14), $zw + \bar z\bar w y$, hence, by (18'), these two exclusions are equivalent to the identity

$$x = z\bar w + \bar z w + \bar z\bar w\bar y, \qquad (d)$$

or

$$\bar x = zw + y\bar z\bar w.$$

ON A NEW ALGEBRA OF LOGIC.

BY O. H. MITCHELL.

THE algebra of logic which I wish to propose may be briefly characterized as follows: All propositions — categorical, hypothetical, or disjunctive — are expressed as logical polynomials, and the rule of inference from a set of premises is: *Take the logical product of the premises and erase the terms to be eliminated.* No set of terms can be eliminated whose erasure would destroy an aggregant term. So far as the ordinary universal premises are concerned, the method will be seen to be simply the negative of Boole's method as modified by Schröder. The reason is, that the terms which the propositions involve are virtually all on the right-hand side of the copula, instead of all on the left-hand side, as in Boole's method.

Attention is especially called to the treatment here given of particular propositions (of which there is introduced a variety of new kinds) which is homogeneous with that of universals, the process of elimination being precisely the same in each case. For the sake of clearness it may be well to state at the outset that I use addition in the modified Boolian sense, — that is, $x + y$ = all that is either x or y.

second relation means, "if $U = F$ for all or some U, then $U = \bar{m} + F$ for all or some U," and the result is obtained by adding $\bar{m}$ to both sides, remembering that $U + \bar{m} = U$. We have, of course,

$$(\bar{m} + F)_e = (\bar{m} + mF)_e = (m = mF)_c.$$

I now give the solution of the well-known problem of Boole, "Laws of Thought," p. 146. The premises are, remembering that $(a = b) = (\bar{a} + b)_1 (a + \bar{b})_1$,

$$(x + z + vy\bar{w} + vw\bar{y})_1,$$

$$(v + \bar{x} + \bar{w} + yz + \bar{y}\bar{z})_1,$$

$$(\bar{x} + \bar{v}\bar{y} + w\bar{z} + \bar{w}z)_1(xy + vx + wz + \bar{w}\bar{z})_1.$$

Multiplying the premises together, and dropping v from the result, we get

$$(wx\bar{z} + w\bar{x}z + \bar{w}xz + \bar{w}x\bar{y} + \bar{w}\bar{x}y\bar{z})_1, = \text{say } F_1.$$

The four results asked for by the problem are

(1) $$(\bar{x} + w\bar{z} + \bar{w}z + \bar{w}\bar{y})_1$$

(2) $$(w\bar{z} + wz + \bar{w}z + \bar{w}\bar{y} + \bar{w}y\bar{z})_1, \text{ i. e. } (U)_1$$

(3) $$(\bar{y} + \bar{w}\bar{x}\bar{z} + \bar{w}xz + w\bar{x}z + wx\bar{z})_1$$

(4) $$(\bar{w}x + \bar{w}\bar{z} + x\bar{z} + w\bar{x}z)_1.$$

The first gives the predicate of x in terms of y, z, w, being the same as $x \prec w\bar{z} + \bar{w}z + \bar{w}\bar{y}$, and is obtained by adding $\bar{x}$ to F and reducing. The second is the relation among y, z, w, and is obtained by dropping x from F and reducing. The result $(U)_1$ shows that no relation is implied among y, z, w alone. The third gives the predicate of y in terms of x, z, w, and is obtained by adding $\bar{y}$ to F and reducing. The fourth is the relation implied among x, z, w, and is obtained by dropping y from F and reducing. The relation (3) is not in its simplest form, since the implied relation (4) among x, z, w

Boole's logical Calculus and the Concept-script[1]

[1880/81]

In his writings, Leibniz threw out such a profusion of seeds of ideas that in this respect he is virtually in a class of his own. A number of these seeds were developed and brought to fruition within his own lifetime and with his collaboration, yet more were forgotten, then later rediscovered and developed further. This justifies the expectation that a great deal in his work that is now to all appearance dead and buried will one day enjoy a resurrection. As part of this, I count an idea which Leibniz clung to throughout his life with the utmost tenacity, the idea of a *lingua characterica*, an idea which in his mind had the closest possible links with that of a *calculus ratiocinatur*. That it made it possible to perform a type of computation, it was precisely this fact that Leibniz saw as a principal advantage of a script which compounded a concept out of its constituents rather than a word out of its sounds, and of all hopes he cherished in this matter, we can even today share this one with complete confidence. I will quote just the following from the relevant passages:

'Si daretur vel lingua quaedam exacta, vel genus scripturae vere philosophiae, . . . omnia quae ex datis ratione assequi, inveniri possent quodam genere calculi, perinde ac resolvuntur problemata arithmetica aut geometrica.'*

---

* De Scientia universali seu calculo philosophico.

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[1] In 1881, this article was submitted by Frege in turn to the *Zeitschrift für Mathematik und Physik*, the *Mathematischen Annalen* and the *Zeitschrift für Philosophie und philosophische Kritik*, but was in every case rejected by the editors. It finally remained unpublished.

From the report of H. Scholz and F. Bachmann: *Der wissenschaftliche Nachlass von Gottlob Frege* (Paris 1936) we learn that the lost original was 'a manuscript prepared for publication of 103 closely written sides of quarto'. Scholz and Bachmann mention besides that Frege also submitted the manuscript to R. Avenarius for the *Vierteljahrsschrift für wissenschaftliche Philosophie*. However it could be that what Frege submitted to Avenarius was the essay published in this volume on pp. 47 ff. 'Boole's logical Formula-language and my Concept-Script', since Avenarius in his letter to Frege of 20/4/1882 cites the title of the manuscript returned by him as 'Boole's logical Formula-Language'.

The article can scarcely have been composed before 1880, the year in which the review by Schroder mentioned on p. 11 appeared. A great number of the reflections

In Schröder's formulation, the problem is as follows. Suppose we observe a class of phenomena (natural kinds or artefacts, e.g. substances) and arrive at the following general results:

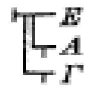
(α) If the characteristics or properties A and C are simultaneously absent from any of the phenomena, the property E is found together with either the property B or the property D but not both.
(β) Wherever A and D are found together in the absence of E, B and C are either both present or both absent.
(γ) Wherever A is found together with B or E or both, either C or D is to be found but not both. And conversely wherever one of C, D is found without the other, then A is to be found together with B or E or both.

We now have to find:

(1) What in general can be inferred about B, C and D from the presence of A,
(2) Whether any relations whatever hold between the presence or absence of B, C and D independently of the presence or absence of the remaining properties,
(3) What follows for A, C and D from the presence of B,
(4) What follows for A, C and D considered in themselves.

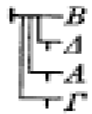
In the solution I use the corresponding Greek capitals so that e.g. A means the circumstance that the property A is to be found in the object under consideration.
I first translate the individual data.

(α) The denial of A and Γ has a consequence the affirmation of E (1).



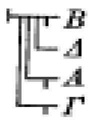
(1)

The denial of A and Γ has as a consequence the affirmation of one of the two B or Δ (2);



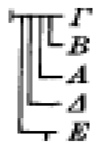
(2)

but it is impossible to have B and Δ together with the denial of A and Γ (3).



(3)

(β) If A and Δ are both to be affirmed and E denied, B and Γ are either both to be affirmed or both denied; that is, if B is affirmed, Γ is also to be affirmed (4);



(4)

STUDIES AND EXERCISES

IN

FORMAL LOGIC,

INCLUDING

A GENERALISATION OF LOGICAL PROCESSES IN THEIR APPLICATION TO COMPLEX INFERENCES.

BY

JOHN NEVILLE KEYNES, M.A.,

LATE FELLOW OF PEMBROKE COLLEGE, CAMBRIDGE.

London:

MACMILLAN AND CO.

1884

properties B, C, and D, will all be wanting. And conversely, *where either the property C or the property D is found singly, or the properties B, C, and D, are together missing, there the property A will be found.*

[Boole, *Laws of Thought*, pp. 146—148. Cp. also Venn, *Symbolic Logic*, pp. 280, 281.]

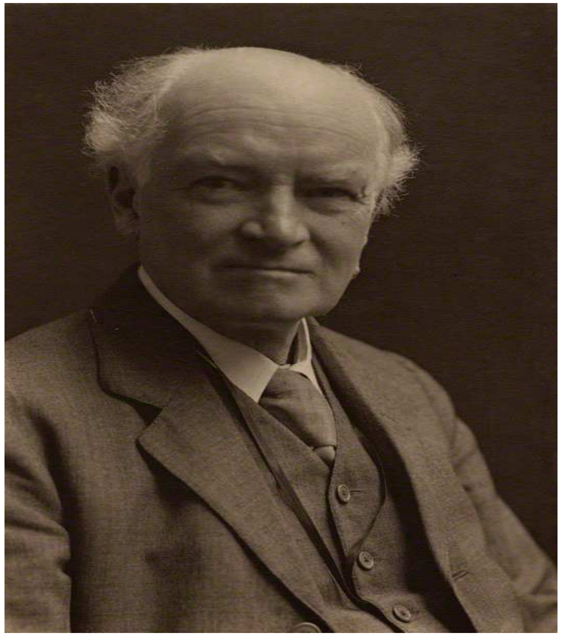
The premisses are as follows:—

1st, All ac is BdE or bDE; (i)
2nd, All ADe is BC or bc; (ii)
3rd, All AB is Cd or cD; (iii)
All AE is Cd or cD; (iv)
All Cd is AB or AE; (v)
All cD is AB or AE. (vi)

We are required to prove:—

All A is Cd or cD or bcd; (α)
All Cd is A; (β)
All cD is A; (γ)
All bcd is A. (δ)

First, By (iii) and (iv), If A is B or E it is Cd or cD;
therefore, A is Cd or cD or be. (1)
By (ii), Ae is either BC or bc or d;
therefore, Abe is bc or d;
therefore, Abe is bce or bde. (2)
By (v), Cd is B or E;
therefore, C is B or D or E;
therefore (by contraposition), bde is c;
therefore, bde is bcd;
therefore, If Abe is bde it is bcd. (3)
Again by (vi), cD is B or E;
therefore (as above), bce is d;
therefore, If Abe is bce it is bcd. (4)



‘In part IV, which contains a generalisation of logical processes in their application to complex inferences, a somewhat new departure is taken. So far as I am aware this constitutes the first systematic attempt that has been made to deal with formal reasonings of the most complicated character without the aid of mathematical symbols and without abandoning the ordinary non-equational or predicative form of proposition. In this attempt I have met with greater success than I had anticipated; and I believe that the methods which I have formulated will be found to be as easy of application and as certain in obtaining results as the mathematical, symbolical, or diagrammatic methods of Boole, Jevons, Venn and others’

(J. N. Keynes, *Formal Logic*, 1884: preface - vii)

- Boole introduced the problem of elimination

- Elimination was considered as the main problem of symbolic logic

- Boole, his followers *and their opponents* worked on its solution

- Symbolic logicians didn't just use Boole's work, they mostly worked the *way* Boole did (a practice, not just a result)

- Symbolic logicians engaged in a 'friendly' competition, for which they invented new notations, new methods (new systems)

- Boole initiated what might be called a 'research program'

Conclusion
Toward Symbolic Logic

Preface

The *Formulaire* aims at publishing propositions that are known in several subjects of mathematical sciences. These propositions are expressed as formulae with the notations of mathematical logic […]

Every part of the *Formulaire*, though started by one author, will be eventually the result of the work of all collaborators.'

(G. Peano, *Formulaire de Mathématiques*, 1895)

FORMULAIRE DE MATHÉMATIQUES

publié par la

" RIVISTA DI MATEMATICA "

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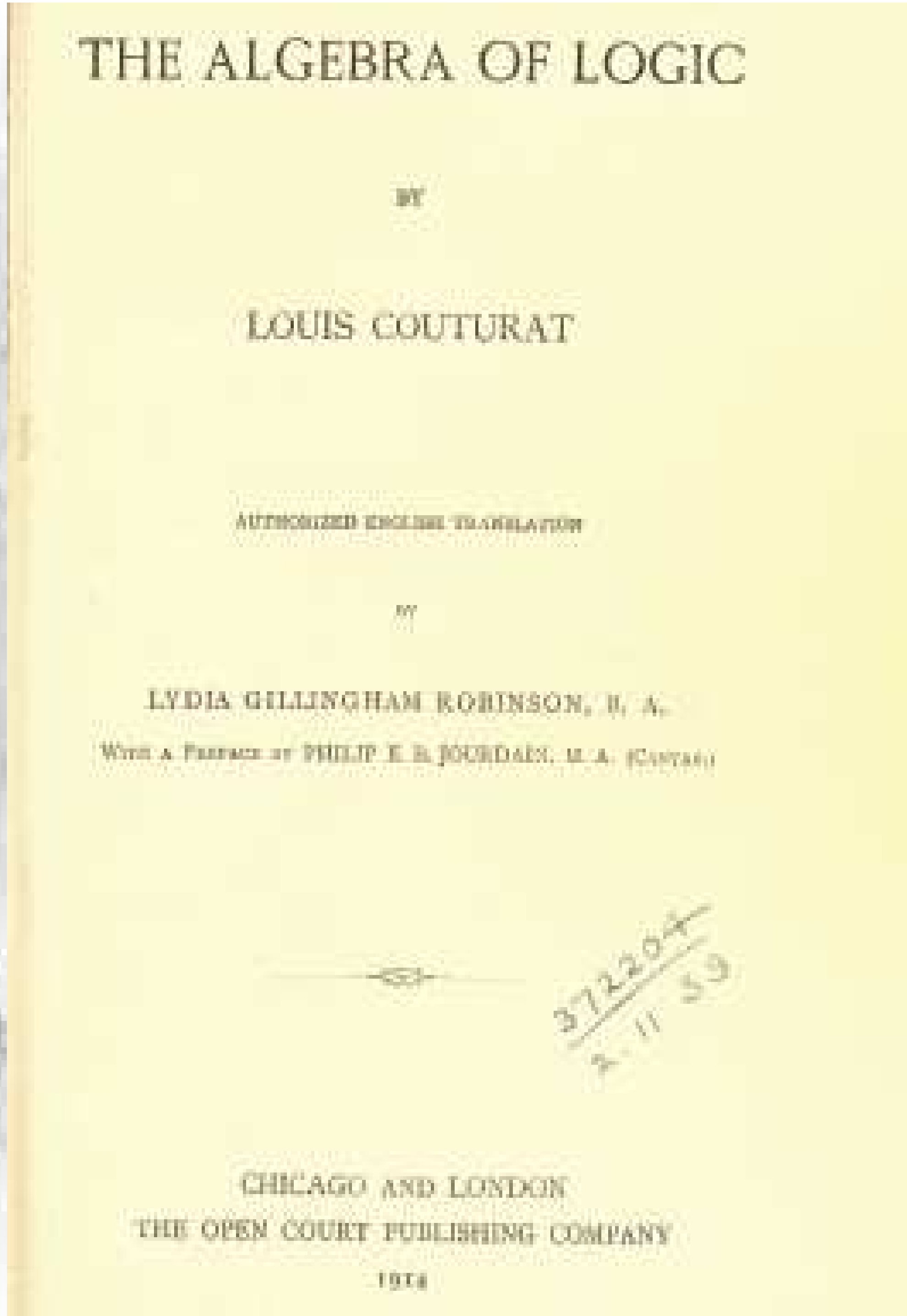
Garbieri, VIII § 6 P5-9.

First International Congress of Philosophy (Paris, August 1900)

- G. Vailati, *'The classification of the sciences'*
- B. Russell, *'The idea of order and absolute position in space and time'*
- H. MacColl, *'Symbolic logic'*
- G. Peano, *'Mathematical definitions'*
- C. Burali-Forti, *'The different logical methods for the definition of the real number'*
- A. Padoa, *'Essay at an algebraic theory of integral numbers, with a logical introduction to any deductive theory whatever'*
- M. Pieri, *'On geometry considered as a pure by logical system'*
- P. Poretsky, *'The theory of logical equalities with three terms.'*
- E. Schröder, *'An extension of the idea of order'*
- W. E. Johnson, *'The theory of logical equations'*
- A. Macfarlane, *'The ideas and principles of the geometric calculus'*



THE ALGEBRA OF LOGIC

BY

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AUTHORIZED ENGLISH TRANSLATION

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THE DEVELOPMENT

OF

SYMBOLIC LOGIC

A CRITICAL-HISTORICAL STUDY OF THE LOGICAL CALCULUS

BY

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PREFACE

The form that the present work has taken is due to some correspondence that I had with Mr. W. E. Johnson in the year 1903. He pointed out to me the error of thinking of the various symbolic systems as being radically distinct, and as competing with one another for general acceptance. Rather, he held, it is correct to adopt the view that there is available at the present time what may be called *the* Logical Calculus, and that towards the creation of this Calculus most symbolists have contributed.

This idea has been worked out in the following pages. I have traced the growth of the subject from the time when Boole originated his generalisations to the time when Mr. Russell, pursuing for the most

Thank you!

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(e.g. the Middle Term of a Syllogism), which I call the "Eliminands," and those which *cannot*, which I call the "Retinends"; and I do not call the Conclusion *complete*, unless it states *all* the relations, among the Retinends only, which can be deduced from the Premisses.

1.

[N.B. In this Problem, it is assumed that the Proposition "All x are y" is equivalent to "Some x are y, and none are y'." See p. 33.]

All the boys, in a certain School, sit together in one large room every evening. They are of no less than *five* nationalities—English, Scotch, Welsh, Irish, and German. One of the Monitors (who is a great reader of Wilkie Collins' novels) is very observant, and takes MS. notes of almost everything that happens, with the view of being a good sensational witness, in case any conspiracy to commit a murder should be on foot. The following are some of his notes:—

(1) Whenever some of the English boys are singing "Rule Britannia", and some not, some of the Monitors are wide-awake;
(2) Whenever some of the Scotch are dancing reels, and some of the Irish fighting, some of the Welsh are eating toasted cheese;
(3) Whenever all the Eleven are oiling their bats, none of the Germans are playing chess;
(4) Whenever some of the Monitors are asleep, and some not, some of the Irish are fighting;
(5) Whenever some of the Germans are playing chess, and none of the Scotch are dancing reels, some of the Welsh are *not* eating toasted cheese;
(6) Whenever some of the Scotch are *not* dancing reels, and some of the Irish *not* fighting, some of the Germans are playing chess;
(7) Whenever some of the Monitors are awake, and some of the Welsh are eating toasted cheese, none of the Scotch are dancing reels;

[TURN OVER.

(8) Whenever some of the Irish are *not* fighting, and some of the Welsh are *not* eating toasted cheese, some of the Eleven are oiling their bats;
(9) Whenever some of the Scotch are *not* dancing reels, and some of the Germans are playing chess, some of the English are *not* singing "Rule Britannia";
(10) Whenever some of the English are singing "Rule Britannia", and some of the Monitors are asleep, some of the Irish are *not* fighting;
(11) Whenever some of the Monitors are awake, and some of the Eleven are *not* oiling their bats, some of the Scotch are dancing reels;
(12) Whenever some of the English are singing "Rule Britannia", and some of the Scotch are *not* dancing reels, * * * *

Here the writer was interrupted, and the MS. breaks off suddenly. The Problem is to complete the sentence, if possible.

2.

(1) A logician, who eats pork-chops for supper, will probably lose money;
(2) A young man always gets up at 5 a.m., unless he has lost money;
(3) No earnest man, who does not eat pork-chops for supper, need take to cab-driving, unless he gambles;
(4) A logician, who is in danger of losing money, had better take to cab-driving;
(5) A gambler, whose appetite is not ravenous, will probably lose money;
(6) A man who is depressed, having lost money and being likely to lose more, always rises at 5 a.m.;
(7) A man, who neither gambles nor eats pork-chops for supper, is sure to have a ravenous appetite;
(8) A lively man, who goes to bed before 4 a.m., had better take to cab-driving;
(9) A man with a ravenous appetite, who has not lost money and does not rise at 5 a.m., always eats pork-chops for supper;

- *The problem of the School-Boys* (12 premisses, 14 terms)
- L. Carroll, *A Challenge to Logicians*, 1892

"Over the past ten years, in three universities in Britain and America, I have in vain asked logicians of high distinction to solve this problem. Even when I gave them Carroll's own solution and asked them to test the argument of correctness, they still tended to scamper off like white rabbits, even though the latter was a task for which their training had prepared them. Occasionally they would counterattack, and demand an explanation of my "antiquarian interest."

(W. W. Bartley III, *Lewis Carroll's Symbolic Logic*, 1977: 25)

‘Bartley has been surprised to find that eminent modern logicians did not accept his challenge to tackle one of the problems in predicate logic set by Lewis Carroll who had techniques and an acquired knack for solving such problems. Bartley does not realize that such skills have no more to do with serious logic than a calculating boy’s ability to raise 123,456,789 to a high power in his head has to do with number theory.’

(P. Geach, 1978: 124)

Is the problem of elimination …

- Artificial?

[So what?]

- Not serious?

[What matters is not the challenge but the nature of the problem]

- Modern?

[Switching theory]