Titles and Abstracts

Jonathan Bennett (Birmingham)

Title: Subdyadic Littlewood-Paley theory and applications

Abstract: We study variants of the classical Littlewood-Paley g-functions which decouple frequency decompositions at scales finer than dyadic. As an application we prove weighted norm inequalities for classes of highly oscillatory convolution operators that fall well beyond the scope of the Calderon-Zygmund theory. This is joint work with David Beltran and Teresa Luque.

Anthony Carbery (Edinburgh)

Title: The Nonlinear Loomis–Whitney inequality

Abstract: We give a simple proof of the Bennett–Carbery–Wright nonlinear Loomis–Whitney inequality using the theory of multilinear duality and factorisation recently developed by Carbery and Valdimarsson.

Michael Christ (Berkeley)

Title: On the fine structure of the Fourier transform and additive combinatorics

Abstract: One of the most fundamental facts concerning the Fourier transform is the Hausdorff-Young inequality, which states that for any locally compact Abelian group, the Fourier transform maps L^p boundedly to L^q, where the two exponents are conjugate and $p \in [1, 2]$. For Euclidean space, the optimal constant in this inequality was found by Beckner for general exponents. Lieb later determined that all extremizers are Gaussian functions. This is a uniqueness theorem; these Gaussians form the orbit of a single function under the group of natural symmetries of the inequality.

We have established a stabler form of uniqueness for $1 < p < 2$: If a function f nearly achieves the optimal constant in the inequality, then f must be close in L^p norm to a Gaussian, with a quantitative bound proportional to the square of the distance to the nearest Gaussian. This can be equivalently viewed as a refinement of the inequality.

The analyses of Beckner and Lieb rely on functorial properties of the Hausdorff-Young inequality which are absent for closely related inequalities. For instance, consider the L^q norm of the Fourier transform of the indicator function of a set in $\mathbb{R}^d$. Among sets of prescribed Lebesgue measure, which have the largest Fourier transforms in this sense? We have obtained some partial results, showing that for all exponents in certain ranges, maximizing sets are ellipsoids.

Common to the analyses of both problems are precompactness theorems, which guarantee that extremizers exist, and that functions or sets that nearly extremize the inequalities must be close to exact extremizers. I will discuss elements of the proofs, at the heart of which lie principles of additive combinatorics which complement structural properties of the Fourier transform.

Anthony Dooley (Bath)

Title: Orbital Convolution Theory on Lie Groups and Symmetric Spaces

Abstract: This geometric approach to harmonic analysis on Lie groups and symmetric spaces derives from the Kirillov orbit method. The aim is to find essentially Euclidean tools to describe non-commutative harmonic analysis.

Taryn Flock (Birmingham)

Title: Regularity of the Brascamp-Lieb Constant

Abstract: The Brascamp-Lieb inequality generalizes many important inequalities in analysis, including the Hölder, Loomis-Whitney, and Young convolution inequalities. Sharp constants for such

inequalities have a long history and have only been determined in a few cases. We investigate the stability and regularity of the sharp constant as a function of the implicit parameters. The focus of the talk will be a local-boundedness result with implications for certain nonlinear and combinatorial generalizations. This is a preliminary report of joint work with Jonathan Bennett, Neal Bez, and Sanghyuk Lee.

Allan Greenleaf (Rochester)

Title: Restricted linear convolution inequalities

Abstract: One of the basic inequalities of harmonic analysis is Young's inequality for the L^r norm of the convolution of two functions on $\mathbb{R}^d$, controlled by their L^p and L^q norms. In this spirit, we prove L^r space estimates for restrictions of convolutions to linear subspaces, controlled by certain mixed norms of the convolution factors, defined in terms of the Fourier transform. Applications include estimates for multi-linear convolution operators, estimates for products of solutions to heat and wave equations, and restrictions of Fourier transforms to *linear* submanifolds. This is joint work with D. Geba, A. Iosevich, E. Palsson and E. Sawyer.

Philip Gressman (Pennsylvania)

Title: Geometric convolutions and Fourier restriction beyond curves and hypersurfaces

Abstract: I will present recent results relating to two problems in Fourier analysis, L^p-improving properties of convolutions with singular measures and the Fourier restriction problem, both of which deal with the analysis of operators associated to submanifolds of Euclidean space. In both cases the theory is much more well-developed for curves and hypersurfaces than it is for submanifolds of intermediate dimension. This relative lack of positive results is due in part to the problem that the Phong-Stein nonvanishing rotational curvature condition is frequently impossible to satisfy for surprisingly deep algebraic reasons. I will focus primarily on the case of 2-surfaces in $\mathbb{R}^5$, which does not fit nicely into previously-existing combinatorial strategies, and will present a new approach with the potential to apply to a broad range of new cases.

Jonathan Hickman (Edinburgh)

Title: Harmonic Analysis over local fields: Part II

Abstract: (Common with James Wright) Recently substantial progress has been made on several major open problems in harmonic analysis by considering discrete models formulated over finite fields. In particular, Wolff's finite field Kakeya conjecture was famously resolved by Dvir; the techniques introduced in this proof were then developed by Guth et al to study the original Euclidean Kakeya and restriction conjectures. Finite fields do not encapsulate all aspects of analysis over $\mathbb{R}$, however, and the lack of any non-trivial notion of scale in the former leads to divergence between the two theories.

One approach to modelling multiple scales in a discrete setting is to work over the ring of integers $\mathbb{Z}/p^k\mathbb{Z}$ (or more generally, $\mathbb{Z}/N\mathbb{Z}$ for composite N). In this case, each 'scale' corresponds to a possible number of divisors and the resulting theory closely parallels that of the Euclidean case. This in turn naturally leads one to consider analysis over non-archimedean local fields (such as the p-adics or p-series fields), a topic which already has a long history dating back to the 1960s and efforts to generalise classical Calderón-Zygmund theory.

In these talks we will develop some modern aspects of harmonic analysis over local fields, such as estimates for oscillatory integrals and Kakeya-type theorems. We will draw particular attention to how the algebraic structure of the field can be used to effectively model familiar localisation and

discretisation phenomena from the Euclidean setting.

Marina Iliopoulou (Birmingham)

Title: Kakeya-type questions and the polynomial method

Abstract: A Kakeya set in $\mathbb{R}^n$ is a subset of $\mathbb{R}^n$ containing a unit line segment in each direction. It is conjectured that such sets have full Hausdorff dimension. Even though the conjecture is far from resolved, discrete analogues have been understood via underlying algebraic hypersurfaces (techniques originating in the work of Dvir, Guth and Katz), in ways which in turn suggest possible ways to approach the original problem. After a discussion on this, we will see how such polynomial techniques are used in discrete settings.

Victor Lie (Purdue)

Title: Extremizers for the 2D Kakeya problem

Abstract: Let Q_0 be the unit square and let $\mathbf{T}$ be a collection of M^{-2n} separated tubes inside Q having length one and width M^{-2n} for some large $M \in \mathbb{N}$. Assume that $\mathbf{T} = \mathbf{T}_1 \cup \mathbf{T}_2$ with $\mathbf{T}_1$ consisting of tubes that have slopes between $[0, \frac{1}{10}]$ and $\mathbf{T}_2$ having tubes with slopes in $[\frac{9}{10}, 1]$. Our goal is to understand *both the structure and the size* of the level sets

$$\{F > \alpha\}$$

where $\alpha > 0$ and $F := (\sum_{\tau\in\mathbf{T}_1} \chi_\tau)(\sum_{\tau\in\mathbf{T}_2} \chi_\tau)$ stands for the bilinear Kakeya function.

Specifically, the main interest is for the situation $\epsilon \in (0,1)$ fixed small number such that

$$\#\mathbf{T}_1 \approx \#\mathbf{T}_2 \approx M^{2n} \text{ and } \alpha \in [M^{2(1-\epsilon)n},\, M^{2(1+\epsilon)n}] \tag{1}$$

From just Chebyshev's inequality we know that

$$|\{F > \alpha\}| \lesssim \frac{1}{\alpha}\,,$$

and we also know that this upper bound is attained when all the tubes are focussing in a square region of side-length $\frac{1}{\sqrt{\alpha}}$. Informally, our goal is to say that this is the "only" geometric instance is which this upper bound can be attained thus characterizing the extremizers for the bilinear Kakeya function.

Our analysis will involve additive combinatorics (e.g. Plnnecke sum-set estimate) and incidence geometry (e.g. Szemeredi-Trotter inequality) techniques and relates with a class of problems including Bourgain's sum-product theorem and Katz-Tao ring conjecture.

This is a joint work with Michael Bateman.

Neil Lyall (Georgia-Athens)

Title: Embedding simplices in sets of positive upper density

Abstract: We will discuss some results pertaining to the embedding of simplices in subsets of $\mathbb{R}^d$ of positive upper density. Time permitting we will also discuss the analogous problem from subsets of $\mathbb{Z}^d$. Joint work with Lauren Huckaba and Akos Magyar.

Alessio Martini (Birmingham)

Title: L^p functional calculus for the Kohn Laplacian on forms on complex spheres

Abstract: The Kohn Laplacian L associated to the tangential Cauchy-Riemann complex on a strictly pseudoconvex CR manifold M is a classic example of a non-elliptic, hypoelliptic differential operator. In the case M is the unit sphere in $\mathcal{C}^n$, a great amount of information about the spectral theory of L can be obtained via representation theory of unitary groups. Following this approach, in joint work (arXiv:1501.02321) with V. Casarino (Padova), M.G. Cowling (UNSW Sydney), and A. Sikora (Macquarie Sydney), we prove a multiplier theorem of Mihlin-Hormander type for L that improves previously known results. In particular, we require on the multiplier a smoothness condition of order $s > (2n-1)/2$, i.e., half the topological dimension of M. It is still an open question whether the same result can be obtained for an arbitrary compact strictly pseudoconvex CR manifold M.

Mariusz Mirek (Bonn)

Title: $\ell^p(\mathbf{Z}^d)$ boundedness for discrete operators of Radon types: maximal and variational estimates.

Abstract: In recent times - particularly the last two decades - discrete analogues in harmonic analysis have gone through a period of considerable changes and developments. This is due in part to Bourgain's pointwise ergodic theorem for the squares on $L^p(X, \mu)$ for any $p > 1$. The main aim of this talk is to discuss recent developments in discrete harmonic analysis. We will be mainly concerned with $\ell^p(\mathbf{Z}^d)$ estimates ($p > 1$) of r-variations ($r > 2$) for discrete averaging operators and singular integral operators along polynomial mappings. All the results are subjects of the ongoing projects with Elias M. Stein and Bartosz Trojan.

Detlef Müller (Kiel)

Title: Necessary and sufficient conditions for L^p spectral multipliers on 2-step groups

Abstract: Let G be a 2-step stratified group of topological dimension d and homogeneous dimension Q. Let L be a homogeneous sub-Laplacian on G. By a theorem due to Christ and to Mauceri and Meda, an operator of the form $F(L)$ is of weak type $(1,1)$ and bounded on $L^p(G)$ for all $p \in (1,\infty)$ whenever the multiplier F satisfies a scale-invariant smoothness condition of order $s > Q/2$. It is known that, for several 2-step groups and sublaplacians, the threshold $Q/2$ in the smoothness condition is not sharp and in many cases it is possible to push it down to $d/2$. Here we show that, for all 2-step groups and sublaplacians, the sharp threshold is strictly less than $Q/2$, but not less than $d/2$.

This is joint work with Alessio Martini.

Keith Rogers (ICMAT Madrid)

Title: Uniqueness for the Calderón problem with Lipschitz conductivities

Abstract: We will review recent progress for Calderón's inverse problem in which one hopes to determine the conductivity $\gamma : \Omega \to (0,\infty)$ of a body Ω. In order to do this, voltages are placed on the boundary $\partial\Omega$, and the induced currents, perpendicular to $\partial\Omega$, are measured. In other words, we hope to recover γ from the Dirichlet-to-Neumann-map of the associated conductivity equation. Assuming that the conductivities are Lipschitz, we prove uniqueness in higher dimensions. That is to say, we show that no two Lipschitz conductivities give rise to the same Dirichlet-to-Neumann-map. Our proof builds on the work of Sylvester and Uhlmann, Brown, and Haberman and Tataru who proved uniqueness for C^1-conductivities and Lipschitz conductivities sufficiently close to the identity (as long as $\|\nabla \log \gamma\|_\infty$ is sufficiently small). We will recall their ideas, before sketching the proof of a Carleman estimate that we use in order to remove the smallness condition. This is joint work with Pedro Caro.

Bartosz Trojan (Wroclaw)

Title: ℓ^p-estimates for discrete operators of Radon types

Abstract: We show $\ell^p(\mathbb{Z}^d)$ boundedness, for $p > 1$, of discrete maximal functions corresponding

with truncated singular integrals of Radon types. The talk will be based on a joint work with M. Mirek and E.M. Stein.

James Wright (Edinburgh)

Title: Harmonic Analysis over local fields: Part I

Abstract: (Common with Jonathan Hickman) Recently substantial progress has been made on several major open problems in harmonic analysis by considering discrete models formulated over finite fields. In particular, Wolff's finite field Kakeya conjecture was famously resolved by Dvir; the techniques introduced in this proof were then developed by Guth et al to study the original Euclidean Kakeya and restriction conjectures. Finite fields do not encapsulate all aspects of analysis over $\mathbb{R}$, however, and the lack of any non-trivial notion of scale in the former leads to divergence between the two theories.

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