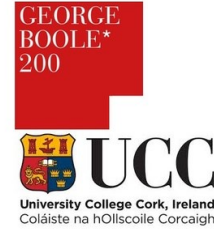


**GBMS Theme 2: From Boole's Algebra of Logic
to Boolean Algebra, and Beyond**



Symbolic Knowledge in Boole's Algebra of Logic

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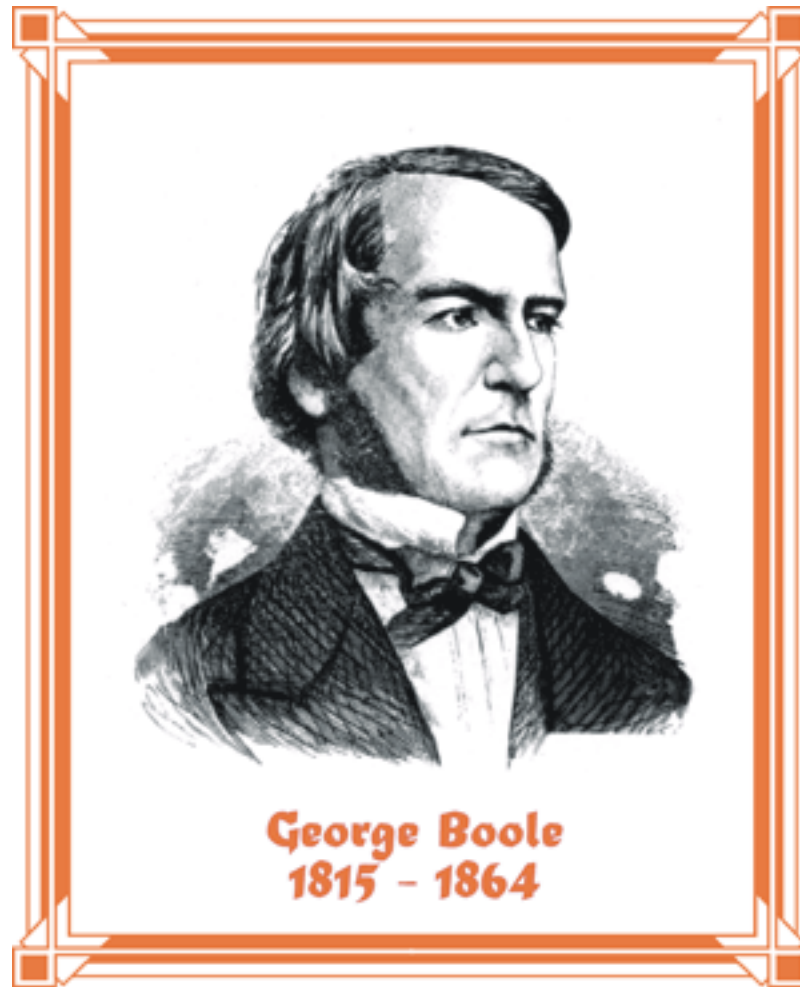
AIMS

The aim of this contribution is to trace and analyze the notion of *symbolic knowledge*, due originally to G. W. Leibniz, that can be found implicitly in George Boole's Algebra of Logic.

Hence, the place of Boole's ideas in the tradition of symbolic knowledge can be shown.

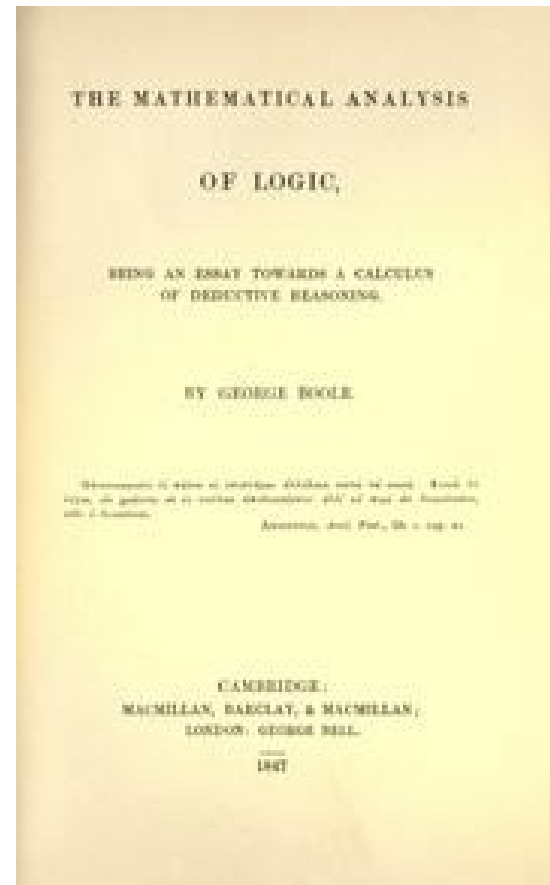
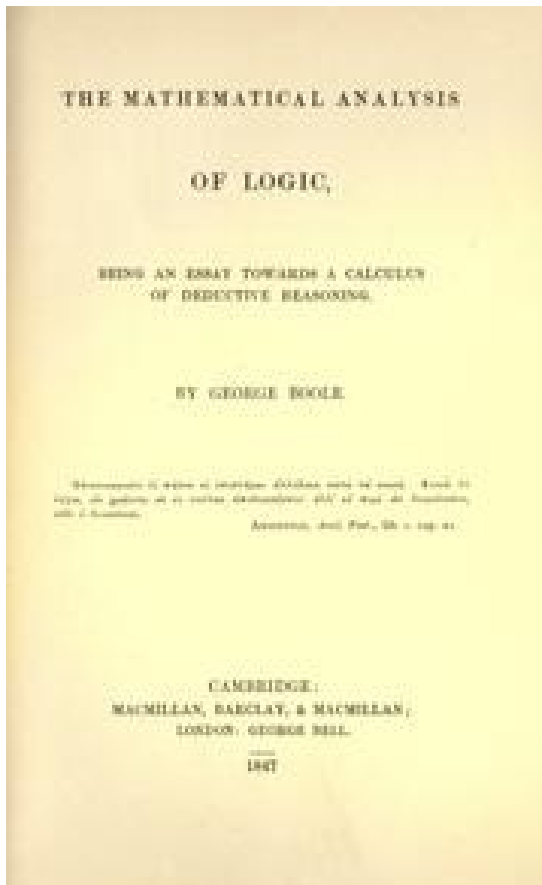
In the talk the presence of different features of symbolic knowledge in Boole's work will be shown.

Boole and Symbolic Knowledge



Boole and Symbolic Knowledge

GEORGE BOOLE: The founder of the algebra of logic



ALGEBRA OF LOGIC

“They who are acquainted with the present state of the theory of Symbolic Algebra, are aware, that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed, but solely upon the laws of their combination. Every system of interpretation which does not affect the truth of the relations supposed, is equally admissible, and it is thus that the same process may, under one scheme of interpretation, represent the solution of a question on the properties of numbers, under another, that of a geometrical problem, and under a third, that of a problem of dynamics or optics. This principle is indeed of fundamental importance”. (Boole 1847, p. 3)

ALGEBRA OF LOGIC

“We might justly assign it [the former principle] as the definitive character of a true Calculus, that it is a method resting upon the employment of Symbols, whose laws of combination are known and general, and whose results admit of a consistent interpretation [...] It is upon the foundation of this general principle, that I purpose to establish the Calculus of Logic.” (Boole 1847, p. 4)

SYMBOLIC KNOWLEDGE



Gottfried Wilhelm von Leibniz
1646 - 1716

SYMBOLIC KNOWLEDGE

symbolic knowledge = knowledge obtained by means of
a **semiotic system**

symbolicus = 'by means of signs (*characteres*)' i.e.
'semiotic'

Boole and Symbolic Knowledge



LEIBNIZ “*Meditationes de cognitione, veritate et ideis*” *Acta Eruditorum*,
November 1684.

SYMBOLIC KNOWLEDGE

“blind thought” (*cogitatio caeca, pensée sourde*),

“blind knowledge” (*cognitio caeca*),

“suppositive notion” (*cognitio suppositiva, connaissance suppositive*)

“suppositive notion” (*notio suppositiva*)

“blind concept” (*conceptus caecus*).

Symbolic knowledge: Reasoning with signs

“The human mind cannot advance too far in *reasoning without resorting to characters*. And characters, when they are adequately chosen, have this marvellous property: they leave so to say *marks of our thoughts on paper*, and [thus] we provide ourselves with the means of being infallible.” (Leibniz, A II, 1, 269-271). (my emphasis)
(Letter to Mariotte from July 1676)

Symbolic knowledge: Reasoning with signs

“...every reasoning is a combination of characters.”

(Leibniz, A VI, 4, 922, ca. 1688-1690)

Symbolic knowledge: formulae and pictures

“in order to think correctly we need some perceptible instruments that I reduce to two main items, *characters* and *tables*...” (Leibniz A VI, 4, 324.)

“...both models and pictures could be included into the characters...” (Leibniz A VI, 4, 324.)

“It is necessary to reduce all sciences to pictures and formulae.” (Leibniz A VI, 4, 439.)

Two features of symbolic knowledge

1. Symbolic Knowledge corresponds to the general notion of *calculus or computation*.
2. Symbolic knowledge fulfils a *surrogative function*.

Two features of symbolic knowledge

1. Signs can be treated as physical systems subjected to operation rules, so that they can be manipulated like other physical objects. Thus, Symbolic Knowledge corresponds to the general notion of *calculus or computation*.

Two features of symbolic knowledge

2. Symbolic knowledge fulfils a *surrogative function*: the consideration of analysis of ideas or things *is replaced or substituted* by our dealing with signs. *Semiotic surrogation* is based on the *analytical design* of complex signs like icons pertaining to systems of signs that are rule-governed and has the general structure of a calculus. The structure of the object or state of affairs *is projected* into the structure of the complex sign.

- Boole and Symbolic Knowledge

Symbolic knowledge in post-Leibnizian thought

- Christian Wolff, Johann Lambert, Kant's pre-critical work.
- French mathematics in 18th C.

SYMBOLIC KNOWLEDGE IN BOOLE'S LOGIC

“[...] the validity of a conclusion arrived at by any symbolical process of reasoning, does not depend upon our ability to interpret the formal results which have presented themselves in the different stages of the investigation.” (Boole 1854, p. 67)

SYMBOLIC KNOWLEDGE IN BOOLE'S LOGIC

“The conditions of valid reasoning, by the aid of symbols, are

1st, That a fixed interpretation be assigned to the symbols employed in the expression of the data; and that the laws of the combination of those symbols be correctly determined from that interpretation.

2nd, That the formal processes of solution or demonstration be conducted throughout in obedience to all the laws determined as above, without regard to the question of the interpretability of the particular results obtained.

3rd, That the final result be interpretable in form, and that it be actually interpreted in accordance with that system of interpretation which has been employed in the expression of the data.” (Boole 1854, p.68)

THE SECOND CONDITION CORRESPONDS TO A “BLIND” PROCEDURE

CATEGORICAL PROPOSITIONS AS EQUATIONS

A: All Xs are Ys	----->	$x(1 - y) = 0$
E: No Xs are Ys	----->	$xy = 0$
I: Some Xs are Ys	----->	$v = xy$
O: Some Xs are not Ys	----->	$v = x(1 - y)$

THE REPRESENTATION OF THE *MODUS BARBARA*

DEDUCTION AS COMPUTATION

$$y (1-x) = 0$$

$$z (1-y) = 0$$

$$z (1-x) = 0$$

CONCLUSIONS I

Boole conceived two basic ways of producing symbolic knowledge:

(a) by **manipulation of signs according to rules** (this way would be knowledge by formal calculus in the stricter sense),

(b) by **application of the sign system to a new domain**, so that new properties of the domain can be known.

In the last case knowledge of formal structures is involved and it could be possible to speak of '**structural knowledge**' as a form of symbolic knowledge.

CONCLUSIONS II

- (1) The algebraic formulation of logic played in Boole a more *pragmatic* than semantic role, namely the solution of logical problems.
- (2) Boole solved logical problems through ‘computations’ in algebra. However, compared with the preceding attempts at logical calculi in the 18th Century, Boole devised an *algebraic structure* for logic, so that a whole new perspective for the analysis of logic opened up.

References

Boole, G. 1847. *The Mathematical Analysis of Logic. Being an Essay Towards a Calculus of Deductive Reasoning*. Cambridge. Macmillan, Barclay and Macmillan

Boole, G. 1854. *An Investigation of The Laws of Thought, on which are Founded The Mathematical Theories of Logic And Probabilities*. London, Walton and Maberly.

Esquisabel, Oscar M. 2012. “Representing and Abstracting. An Analysis of Leibniz’s Concept of Symbolic Knowledge.” In Lassalle Casanave 2012, pp. 1-49.

Lassalle Casanave, Abel (ed.) 2012. *Symbolic Knowledge from Leibniz to Husserl*. London, College Publications 2012.

Legrís, Javier. 2012. “Between Calculus and Semantic Analysis. Symbolic Knowledge in the Origins of Mathematical Logic”. In Lassalle Casanave 2012, pp. 79-113.

Thank you!

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