A brief guided tour along the borders of Logic Programming with Description Logics

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Overview

Motivation

Rules complement and extend ontologies

- Rules are a powerful way of modeling knowledge
- Rules have been successfully applied in Logic Programming (LP) and Databases [Ceri et al., 1990]
- Rules play also a role in the *Semantic Web* [Horrocks et al., 2003a]
  - See the Rules Interchange Format (RIF) activity at W3C

Rule acquisition needs automation

- Demanding knowledge engineering activity for very large KBs
  - Rule authoring task more error-prone than rule reviewing
  - Partial automation by applying rule learning algorithms
- Need to take ontologies into account

Description Logics

- Family of logic-based KR formalisms [Baader et al., 2007]
  - Structured incomplete knowledge
  - Focus on entities rather than on relationships
  - Descendants of semantic networks and KL-ONE
- Mapping to decidable FOL fragments [Borgida, 1996]
  - Variable-free syntax
- Killer application: ontologies and Semantic Web

Description Logics I

Basics

Atomic concepts Unary predicates/formulae with one free variable (A)

- e.g., *Person*, *Doctor*, *HappyParent*

Atomic roles Binary predicates/formulae with two free variables (R)

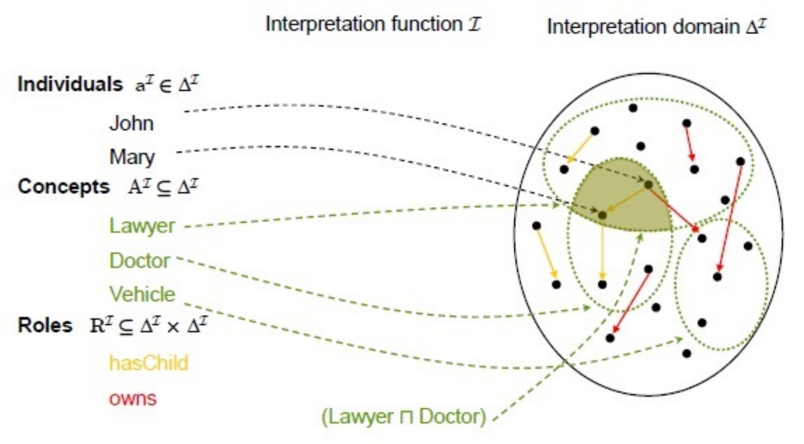
- e.g., *hasChild*, *loves*

Individuals Constants (a)

- e.g., *John*, *Mary*, *Italy*

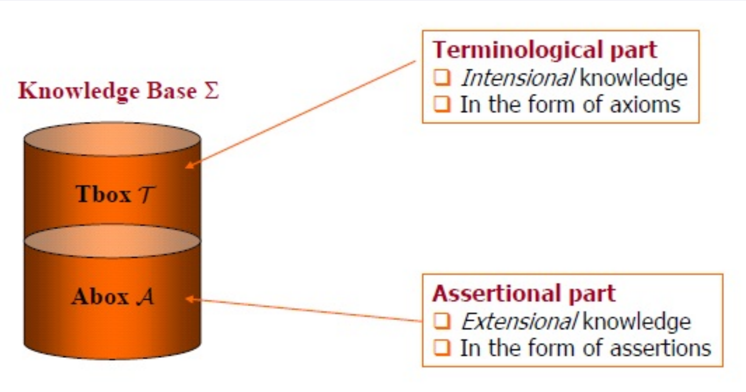
Constructors Operators (*e.g.*, $\sqcap$) for forming complex concepts and roles from atomic ones restricted so that satisfiability/subsumption is decidable and, if possible, of low complexity

Direct semantics



Description Logics III

Knowledge bases



Description Logics IV

Open World Assumption (OWA)

- The information in an Abox is generally considered to be incomplete (*open world*)
- An Abox represents possibly infinitely many interpretations, namely its models
- Query answering requires nontrivial reasoning
- Classical negation!

Description Logics V

Main reasoning tasks

Consistency check Is the KB satisfiable? *i.e.*, is there a model that satisfies both $\mathcal{T}$ and $\mathcal{A}$?

Subsumption check Does $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ hold in all models of $\mathcal{T}$?

Instance check Is $C(a)$ derivable from Σ?

Description Logics VI

Reasoning algorithms based on tableau calculi

Tableau calculus

1. Applies rules that correspond to DL constructors
   - E.g., $John : (Person \sqcap Doctor) \rightarrow_{\sqcap} John : Person$ and $John : Doctor$
2. Stops when no more rules applicable or clash occurs
   - Clash is an obvious contradiction, e.g., $A(x)$, $\neg A(x)$

- Some rules are nondeterministic (e.g., $\sqcup$, $\exists$)
  - In practice, this means *search*
- Cycle check (blocking) often needed to ensure termination

Description Logics VII

Naming conventions for DLs

$\mathcal{F}$	Functional restrictions
$\mathcal{E}$	Full existential qualification
$\mathcal{U}$	Concept union
$\mathcal{C}$	Complex concept negation
$\mathcal{H}$	Role hierarchy
$\mathcal{R}$	Limited complex role inclusion axioms; reflexivity; role disjointness
$\mathcal{O}$	Nominals
$\mathcal{I}$	Inverse properties
$\mathcal{N}$	Number restrictions
$\mathcal{Q}$	Qualified number restrictions

Notable examples of the DL family

- Smallest expressive DL is $\mathcal{ALC}$ [Schmidt-Schauss and Smolka, 1991]
- $\mathcal{SHIQ}$ [Horrocks et al., 2000] is a very expressive DL
  - $\mathcal{S}$ is an abbreviation for $\mathcal{ALC}$ with transitive roles
  - DL underlying the W3C Ontology Web Language OWL [Horrocks et al., 2003b]
- Efficient query answering in DL-Lite [Calvanese et al., 2007]

Description Logics VIII

Syntax and semantics of $\mathcal{ALC}$ KBs

bottom concept	$\bot$	$\emptyset$
top concept	$\top$	$\Delta^{\mathcal{I}}$
atomic concept	A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
role	R	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
individual	a	$a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
concept negation	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
concept conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
concept disjunction	$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
value restriction	$\forall R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \forall y\ (x,y) \in R^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}}\}$
existential restriction	$\exists R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y\ (x,y) \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
equivalence axiom	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$
subsumption axiom	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
concept assertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$R(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$

Description Logics IX

Example of an $\mathcal{ALC}$ KB $\mathcal{K}$

A1 $\mathit{DairyProduct} \sqsubseteq \mathit{Product}$

F1 $\mathit{DairyProduct}(\mathit{mozzarella})$

Example of instance check in $\mathcal{ALC}$

Is $\mathit{Product}(\mathit{mozzarella})$ derivable from the KB $\mathcal{K}$? or equivalently, is $\mathcal{K} \cup \{\neg \mathit{Product}(\mathit{mozzarella})\}$ consistent?

Description Logics X

Extensions

- Fuzzy DLs
  - See [Straccia, 2015] for a tutorial lecture
- Probabilistic DLs, *e.g.*, [Lukasiewicz, 2008]
  - See Lamma's invited talk at ICLP 2015 on Thursday morning
- LP and DLs
  - *Focus of this talk!*

Logic Programming and DLs

- What are the differences between LP and DLs?
- Is there an overlap between the two?
- Can we combine them?

Differences between LP and DLs

A summary

1. CWA vs OWA
2. Single vs. multiple models
3. Negation as failure vs. classical negation
4. Strong negation vs. classical negation
5. Treatment of equality
   - Unique Names Assumption (UNA) [Reiter, 1980] might not hold in DLs
6. Existential quantification
   - Recent work on DATALOG$^{\pm}$ [Calì et al., 2009]
   - See Lukasiewicz's invited tutorial at ICLP 2015 on Thursday afternoon
7. Decidability

Intersection between LP and DLs

Exploring the overlap

A partial overlap exists between LP and DLs which allows the extension and/or adaptation of known results in LP to DLs and viceversa

- Horn fragment of DLs
  [Krötzsch et al., 2008, Krötzsch et al., 2013, Krötzsch et al., 2015]
- Efficient DL reasoning with LP
  [Hustadt et al., 2007, Lukácsy and Szeredi, 2009]

Integration between LP and DLs I

Exploiting the power of combination

- Combination is more than the sum of the parts
- Very expressive FOL languages as an outcome
- Solutions to the semantic mismatch between LP and DLs

Integration between LP and DLs II

Approaches

Non-hybrid Combination within a *homogeneous* semantic framework

- description logic programs [Grosof et al., 2003]
- DL-safe rules [Motik et al., 2005]
- Semantic Web Rule Language (SWRL) [Horrocks and Patel-Schneider, 2004]

Hybrid Combination within a *heterogeneous* semantic framework

- *Focus of this talk!*

Integration between LP and DLs III

DL-safety [Motik et al., 2005]

- Syntactic restriction to the form of rules
- Helps avoiding unwanted interactions between the LP and the DL parts

Integration between LP and DLs IV

Schemes for hybrid integration

Loose coupling The LP part and the DL part are treated as separate and independent components

- *e.g.*, dl-programs [Eiter et al., 2008b] rely on safe interface between an ASP engine and a DL reasoner

Tight integration Separation between the two vocabularies but notion of an integrated model which satisfies both parts

- *Focus of this talk!*

Full integration No separation between the two vocabularies

- *e.g.*, MKNF-knowledge bases [Motik and Rosati, 2007]

Tight integration of LP and DLs I

Overview

	CARIN [Levy and Rousset, 1998]	$\mathcal{AL}$-LOG [Donini et al., 1998]	$\mathcal{DL}$+LOG$^{\vee}$ [Rosati, 2006]
DL language	any DL	$\mathcal{ALC}$	any DL
CL language	Horn clauses	DATALOG clauses	DATALOG$^{\vee}$ clauses
DL-safety	no	yes	weakly DL-safe
rule head literals	DL/Horn	DATALOG	DL/DATALOG
rule body literals	DL/Horn	$\mathcal{ALC}$/DATALOG (no roles)	DL/DATALOG
semantics	Herbrand models + DL models	idem	stable models + DL models
reasoning	SLD-resolution + tableau calculus	idem	stable model computation + Boolean CQ/UCQ containment
decidability	only for some instantiations	yes	for all instantiations with DLs for which the Boolean CQ/UCQ containment is decidable
implementation	yes, *e.g.*[Goasdoué et al., 2000]	yes, *e.g.*[Ruckhaus et al., 2006]	unknown

Tight integration of LP and DLs II

Example: a $\mathcal{DL}+\text{LOG}^{\neg\vee}$ KB $\mathcal{K} = (\Sigma, \Pi)$

- The DL component Σ

```
[A1] PERSON ⊑ ∃ FATHER⁻ .MALE
[A2] MALE ⊑ PERSON
[A3] FEMALE ⊑ PERSON
[A4] FEMALE ⊑ ¬MALE
     MALE(Bob)
     PERSON(Mary)
     PERSON(Paul)
     FATHER(John,Paul)
```

- The DATALOG component Π

```
[R1] boy(X) ← enrolled(X,c1,ft), PERSON(X), not girl(X)
[R2] girl(X) ← enrolled(X,c2,ft), PERSON(X)
[R3] boy(X) ∨ girl(X) ← enrolled(X,c3,ft), PERSON(X)
[R4] FEMALE(X) ← girl(X)
[R5] MALE(X) ← boy(X)
[R6] man(X) ← enrolled(X,c3,pt), FATHER(X,Y)
     enrolled(Paul,c1,ft)
     enrolled(Mary,c1,ft)
     enrolled(Mary,c2,ft)
     enrolled(Bob,c3,ft)
     enrolled(John,c3,pt)
```

- Notice that $\mathcal{K} \models_{NM}$ FEMALE(Mary), while $\Sigma \not\models_{FOL}$ FEMALE(Mary).

Inductive Logic Programming and DLs

- ILP in a nutshell
- Learning hybrid rules with ILP

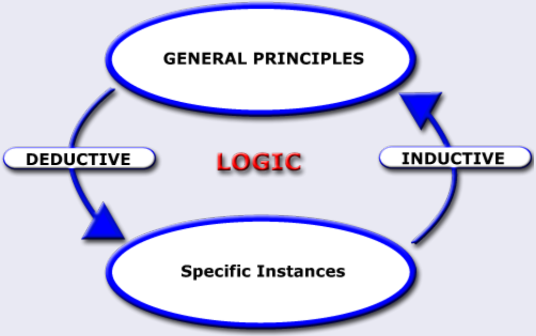
ILP in a nutshell I

Basics [Muggleton, 1990]

- ILP = LP + Concept Learning
  - Concept definitions as logic programs
- Use of background knowledge (BK)
- Bunch of techniques for structuring, searching and bounding the hypothesis space [Nienhuys-Cheng and de Wolf, 1997]

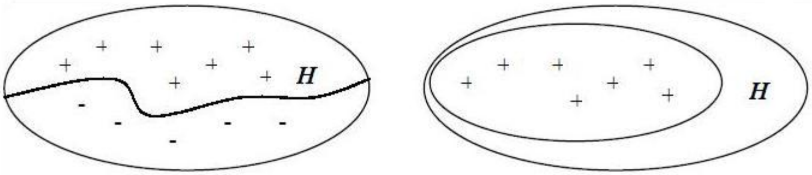
ILP in a nutshell II

Inductive inference



ILP in a nutshell III

Scope of induction



(a) (b)

(a) Discriminate for making predictions on unseen data (*e.g.*, classification)
(b) Characterize for understanding the available data (*e.g.*, clustering)

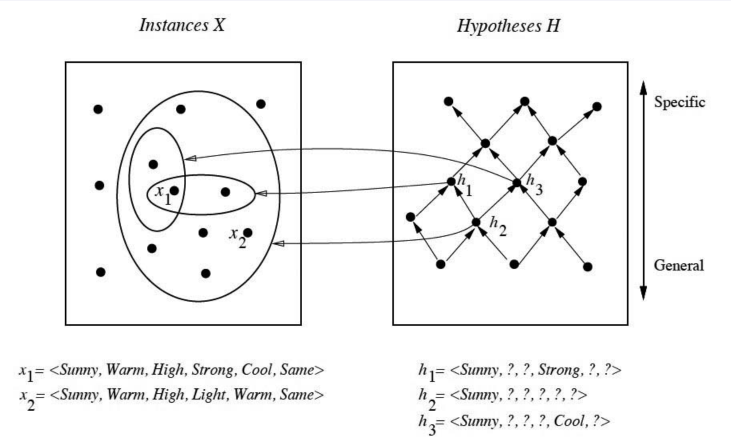
ILP in a nutshell IV

Generalization

- It proceeds from a premise about a sample to a conclusion about the population.
  - The proportion Q of the sample has attribute A.
  - Therefore:
  - The proportion Q of the population has attribute A.
- While the conclusion of a deductive argument is *certain*, the truth of the conclusion of an inductive argument is *probable*, based upon the evidence given.
- Instead of being valid or invalid, inductive arguments are either *strong* or *weak*, which describes how probable it is that the conclusion is true.
  - In a *strong* inductive generalization, the sample must represent the population.

ILP in a nutshell V

Generalization as search [Mitchell, 1982]



ILP in a nutshell VI

Generality orders

Syntactic Clause comparison wrt their structure (*e.g.*, θ-subsumption [Plotkin, 1970])

Semantic Clause comparison wrt their implications (*e.g.*, generalized subsumption [Buntine, 1988])

ILP in a nutshell VII

Extensions

- Probabilistic ILP
  - See [Riguzzi et al., 2014] for a survey
- Concept Learning in DLs
  - See [Lisi, 2013] for a unified view over literature
- Learning hybrid rules
  - *Focus of this talk!*

Learning hybrid rules with ILP I

Overview

	Learning CARIN-$\mathcal{ALN}$ rules [Rouveirol and Ventos, 2000, Kietz, 2003]	Learning $\mathcal{AL}$-LOG rules [Lisi, 2008]	Learning $\mathcal{SHIQ}$+LOG rules [Lisi, 2010]
prior knowledge	CARIN-$\mathcal{ALN}$ KB	$\mathcal{AL}$-LOG KB	$\mathcal{SHIQ}$+LOG KB
ontology language	$\mathcal{ALN}$	$\mathcal{ALC}$	$\mathcal{SHIQ}$
rule language	HCL	DATALOG	DATALOG
hypothesis language	CARIN-$\mathcal{ALN}$ non-recursive rules	$\mathcal{AL}$-LOG non-recursive rules	$\mathcal{SHIQ}$+LOG non-recursive rules
target predicate	Horn predicate	DATALOG predicate	$\mathcal{SHIQ}$/DATALOG predicate
logical setting	interpretations	interpretations/entailment	entailment
scope of induction	prediction	prediction/description	prediction/description
generality order	generalized subsumption	generalized subsumption	generalized subsumption
coverage test	CARIN query answering	$\mathcal{AL}$-LOG query answering	$\mathcal{DL}$+LOG$^{\vee}$ query answering
ref. operators	n.a.	downward	downward/upward
implementation	unknown	yes, see [Lisi, 2011]	no
application	no	yes, see [Lisi and Malerba, 2004]	no

Learning hybrid rules with ILP II

Applications

- Spatial data mining in $\mathcal{AL}$-LOG [Lisi and Malerba, 2004]
- Semantic Web Mining in $\mathcal{AL}$-LOG [Lisi, 2011]
- Learning $\mathcal{DL}$+LOG$^{\vee}$ rules for database design [Lisi, 2010]
- Learning $\mathcal{DL}$+LOG rules for ontology maintenance [Lisi, 2014]

Learning hybrid rules with ILP III

Example: a $\mathcal{DL}+\textsc{log}^{\neg}$ KB $\mathcal{K}$

[A1] $\mathtt{RICH} \sqcap \mathtt{UNMARRIED} \sqsubseteq \exists\, \mathtt{WANTS\text{-}TO\text{-}MARRY}^{-}.\top$

[A2] $\mathtt{WANTS\text{-}TO\text{-}MARRY} \sqsubseteq \mathtt{LOVES}$

[R1] `RICH(X)` $\leftarrow$ `famous(X)`, `not scientist(X)`

[R2] `happy(X)` $\leftarrow$ `famous(X)`, `WANTS-TO-MARRY(Y,X)`

```
UNMARRIED(Mary)
UNMARRIED(Joe)
famous(Mary)
famous(Paul)
famous(Joe)
scientist(Joe)
meets(Mary,Paul,Italy)
meets(Mary,Joe,Germany)
meets(Joe,Mary,Italy)
```

Learning hybrid rules with ILP IV

Example (contd.): What if new knowledge for $\mathcal{K}$ becomes available?

- Let us consider the conceptual changes of $\mathcal{K}$ due to extensional knowledge (*i.e.*, facts of the instance level of the ontology) previously unknown but classified which may become available.
- For example, the new facts `LONER(Joe)`, `LONER(Mary)`, and `LONER(Paul)` concerning known individuals may raise the need for having a definition of the concept `LONER` in the ontology.
- One such definition can be learned from these facts together with prior knowledge about `Joe`, `Mary` and `Paul`.

Learning hybrid rules with ILP V

Example (contd.): learning a rule-based definition of LONER

- Target predicate: LONER
- Examples: $\mathcal{E}^+ = \{\texttt{LONER(Mary)}, \texttt{LONER(Joe)}\}$, $\mathcal{E}^- = \{\texttt{LONER(Paul)}\}$
- Language bias: $\{\texttt{famous/1}\} \cup \{\texttt{happy/1}\} \cup \{\texttt{RICH/1}, \texttt{UNMARRIED/1}\}$
- Candidate hypotheses:

$h_1^{\texttt{LONER}}$	$\texttt{LONER(X)} \leftarrow \texttt{famous(X)}$
$h_2^{\texttt{LONER}}$	$\texttt{LONER(X)} \leftarrow \texttt{famous(X)}, \texttt{UNMARRIED(X)}$
$h_3^{\texttt{LONER}}$	$\texttt{LONER(X)} \leftarrow \texttt{famous(X)}, \texttt{not happy(X)}$

- Valid hypothesis: $h_3^{\texttt{LONER}}$
- Justification: consistency with examples

Conclusions

- LP and the family of DLs are both based on FOL fragments
- Different semantic assumptions
  [Eiter et al., 2008a, Motik and Rosati, 2010, Krötzsch et al., 2015]
  - Partial overlap between the two exists
  - Combination is possible under certain conditions
- Interesting rule learning problems [Lisi, 2014]

Questions?

Thanks for your attention!

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